CHAPTER 2

Groundwork: Political Equality and Political Institutions

This chapter lays out the basic qualities we require for procedures to be called democratic. These qualities are then developed in the next two chapters into a theory of democratic institutions. The remaining chapters consider how institutions can balance the value of democracy so defined with other things we consider valuable, such as rights protection, deliberation, and stability.

We begin with the definition of democracy given by Dahl in *A Preface to Democratic Theory* (1956). A procedure is democratic if it satisfies the properties of popular sovereignty and political equality. Popular sovereignty is satisfied if the people are the final arbiter over every matter, should they choose to be. Thus the people may delegate control over many (indeed most) matters but are always the ultimate authority. Political equality is satisfied when all citizens have equal say on such binding decisions. This is operationalized as everyone's vote being equal, a condition that is harder to meet than it might at first appear. It is not sufficient that everyone have a vote and these votes be equally weighted ("one person, one vote"). It is also necessary that the institutions be so constructed so as not to be biased in favor of any voter (or group of voters) or in favor of any alternative. We will find that this situation is the exception rather than the rule among the countries that are commonly referred to as democracies.

Of course these formal, procedural qualities are not the only criteria we may use to judge how democratic a government is. We may expect a democratic polity to have many other qualities. For example, we may expect it to have high levels of popular participation, a culture favorable to democracy and tolerance, a highly informed populace, a tendency to foster rational social inquiry, a discourse oriented toward impartial rule making rather than special interest pleading, or any number of other qualities. However, formal political equality is more essential, in the sense that it forms a minimal core of what we mean by democracy.

First, it is possible to conceive a government being democratic and lacking the various other qualities listed. For example, it is possible for a

government to be democratic even though it has low participation. We may argue that it is a low-quality democracy, but it is a democracy nevertheless. However, if the institutions are systematically rigged in someone's favor, then the system is no longer democratic. For example, if some people have two votes, and others only one, we would not typically refer to this system of government as democratic. If we have a complicated institutional setup that has the same effect as giving some people two votes, we may refer to it as democratic out of ignorance or hypocrisy. But when it is pointed out that the system effectively gives some people a double vote, we can no longer defend it as democratic, although we may be able to defend it on other grounds, such as the superior competence of some people.

Another way of stating the same point is that a minimal requirement for democracy is that elections be free and fair, and that the decision-making process by elected officials (such as parliamentary rules) also be procedurally fair. This requirement is harder to meet than it may at first appear. When we talk about elections being "free and fair," we often demand a very low standard of fairness. Essentially we simply demand that the election be competitive, and that victory by the government is not preordained (for example, by government monopoly control over the media or outright electoral fraud). As a result, all the countries we refer to as "advanced industrial democracies" easily pass the test, as do many others. This rightly leads to the criticism of the concept of "electoral democracy." However, the response to this criticism should be to define electoral fairness more rigorously, before we search for other requirements. If we take the requirement of electoral fairness seriously (in the manner described later), we will see that this is actually a very demanding requirement that very few of the countries we conventionally describe as democracies come close to meeting. Dismissing the requirement that elections be free and fair as trivial may lead us to ignore the fact that many elections are actually not that fair.

A second reason for concentrating on democratic procedures is that procedures can be legislated, whereas other qualities we view as desirable in a democracy cannot be. Procedures are essentially the rules of the democratic game. Qualities such as the level of participation and the quality of deliberation are outcomes of the democratic process; that is, they are products of the way that people choose to play the democratic game. Whereas we can set up the rules of the game, in general we cannot force people to play the game in a specific way. Indeed, to do so would be self-defeating in a democratic setting. The most we can do is to try to influence people to behave in certain ways, either by moral suasion or by the provisions of incentives (which may include the rules and procedures

themselves). We can design democratic institutions, but we cannot design the demos to order.

A third and final justification for adopting a procedural approach to democracy is that democratic procedure may be complementary to the other qualities we value in democracy. That is to say, adopting democratic procedures may be the most effective way to promote those other values. This, of course, is a strong assumption and will be dealt with in subsequent chapters. For example, chapter 5 will argue that proportional representation elections to a majority-rule parliament is the institutional configuration most likely to protect minority rights, while chapter 6 will make the case that this configuration provides the greatest incentive for democratic deliberation and the rational social inquiry favored by pragmatists in the tradition of Dewey (1946).

However, there is one set of outcomes that is essential to a process being even minimally democratic: the provision of basic rights. As Dahl (1988) argues, certain rights are implicit to the notion of democracy. It makes no sense to talk about democratic procedures if people are unable to make democratic choices, say because of the threat of physical coercion or the inability to speak freely. Furthermore, some other rights that are not implicit in the notion of democracy may in practice be essential to it. Dahl suggests that a degree of economic independence is such a requirement—if one voter was economically dependent on another in a feudal manner, that voter might be unable to choose freely. As Nino (1996, 138) argues, such rights are outcomes of the democratic process, not "things" that exist before it. Constitutions cannot give people rights in a substantive sense. Indeed many authoritarian regimes have had impressive collections of paper rights guaranteed by a constitution that was never enforced.

Thus we are forced to think about rights in a dynamic perspective. The political process in period t protects basic rights that allow democratic choice in period $t + 1$. For a system to remain minimally democratic therefore requires that it be procedurally democratic and that the practice of democracy does not lead to the violation of basic rights that are a condition of future democratic choice. There are different mechanisms by which basic rights can be protected. It can be argued, in the manner of Mill, Dahl, and Rousseau, that the people are the best protectors of their own liberty and thus that democratic procedures are the least likely to infringe on democratic rights. However, there is another tradition that argues that basic rights require protection from the democratic process. As Dahl (1988) points out, this logically requires some degree of guardianship—a nondemocratic body such as a constitutional court has to decide when basic rights have been infringed. This violates

the principle of popular sovereignty but can be defended from a democratic perspective on the grounds that restricting democracy in a few spheres (basic rights) is essential for the maintenance of democracy in all other matters. Alternatively, judicial guardianship can be defended by arguing that the good of defending rights trumps the good of democratic procedure. Chapter 5 considers rights protection in more depth and broadly argues for the democratic view that democratic procedure provides the best defense for democratic rights. However, for current purposes, it is enough to recognize that democratic procedure is a necessary, but not sufficient, condition for democratic government.

Dahl (1956) refers to the theory of democracy based on the axioms of popular sovereignty and political equality as the "populist theory of democracy." After outlining this in chapter 2 of *A Preface to Democratic Theory,* he dismisses it as being of little practical relevance because it is purely axiomatic. In particular the populist theory mandates majority rule, but actual democracies are not direct but representative and so cannot simply use majority rule to make decisions. In place of the populist theory, he outlines a behavioral theory of democracy, which he refers to as polyarchy. This is not, as is sometimes supposed, essentially a description of the American system of government. The "American Hybrid" is dealt with separately in Dahl's chapter 5, and he is explicit that the American system does not meet all of the requirements of polyarchy. Instead polyarchy is an operationalization of the populist theory of democracy laid out in chapter 2.

This chapter and the next two will argue that the axioms of popular sovereignty and political equality can be applied far more directly to political institutions. Furthermore, they can give us very specific prescriptions as to what democratic institutions should look like, as well as specific criteria to judge how democratic the existing institutions are. Contrary to Dahl's assertion, the fact that all modern democracies are representative is not a stumbling block to an axiomatic theory. However, it does force us to separate the democratic process into two sets of procedures. First there are seat allocation rules, whereby the votes of the population are translated into seats in decision-making bodies, such as legislatures or executives. Second, there are social decision rules, by which these bodies make binding decisions. The next section considers this distinction.

Seat Allocation Rules and Social Decision Rules

This book treats seat allocation rules and social decision rules as separate species, to which it is appropriate to apply different standards. This

is in contrast with the direction typically taken by social choice theory. (Austen-Smith and Banks 2005 is a notable exception.) Typically, social choice theory looks for a procedure that takes people's preference orderings as an input and produces as an output an ordering of either social states or candidates for office. This approach is usually justified in either epistemic or constitutive terms. That is, the outcome is taken to be the "will of the people" or the correct decision by definition (constitutive) or to be an indicator of it (epistemic). In either case, the procedure takes voters' orderings of alternatives or candidates and is supposed to produce a social ordering that corresponds to the collective will of the voters or the correct ordering of the alternatives based on the voters' assessment of their merits.

For example, Arrow's (1951/1963) work is built around the concept of a social welfare function, which is defined as a procedure that translates voters' preference orderings into a social ordering. Black (1958/1971) uses a statistical analogy, looking for a procedure that produces a suitable "average" based on voters' ranking of the candidates. However, the justification of this is still to produce an ordering of the candidates based on their merits. Going further back, Borda (1770/1995) justifies his proposed procedure in terms of correctly reflecting the will of the people and in terms of ranking the candidates correctly in terms of their merits. Furthermore he explicitly argues that the same procedure should be applied to the selection of candidates as to the choice of alternatives by a committee. Condorcet (1785/1995) argues that an appropriate voting system is one that maximizes the probability of choosing the worthier competitor or correct alternative based on the pairwise rankings of the candidates or alternatives by the voters.

This approach runs into problems for two reasons. First, as Black (1948, 1958/1971) and Arrow (1951/1963) discovered, it is not possible to find a single function that translates individual orderings into social orderings in a satisfactory way. This has led to the conclusion that democracy is meaningless or at least that it can only be defended in the most minimalist terms, in that it merely ensures that governments can sometimes be removed (as argued most notably in Riker 1982). However, this conclusion depends on the epistemic demand that democratic procedures must reveal the true "will of the people" (that is, a single ordering of social preferences). If instead we simply require that procedures satisfy some requirements of fairness, the skeptical conclusion that democracy is empty proves to be unwarranted. Chapter 4 presents an argument for this view.

Second, actual democratic institutions are not social welfare functions, nor should they be. The underlying normative justifications of actual political institutions and theoretical social welfare functions are

quite different. Seat allocation rules are not justified because they pick the best 150 or 435 or however many candidates. Instead they are justified in terms of producing an assembly that is representative (in some way) of the voters. Social decision rules in practice are not used to determine outcomes from the raw data of legislators' preferences. They are used to structure a deliberative process in which legislators debate and negotiate outcomes. If we are to talk about what institutions can satisfy the procedural demands of democracy (most notably, political equality) we have to start from the classes of procedure we actually use in representative democracy.

Seat allocation rules are used to choose representatives, who subsequently make social decisions. Given that choice of the representatives is not the final outcome, it makes little sense to talk about the procedure choosing the 150 or however many "best" candidates in any kind of ultimate sense. Rather the goal is to produce an assembly that is representative of the voters. Given that it is impractical for all voters to take part in the final deliberative process, it is necessary for someone to be present for (re-present) them. Of course, there is a tremendous amount of ambiguity in terms of what is meant by *representation*. It could be meant in terms of shades of opinion, or demographic characteristics, or geographical residence, or some other criterion. The most frequently cited work on this (Pitkin 1967) gives a rich typology of conceptions of representation but no definitive conclusion. Nevertheless, it is clear that the process of representation is to some degree distributive, that is, about sharing legislative seats among different groups of people, not simply about choosing the best candidates. This is most obvious in a proportional representation system, where the legislative seats are distributed in proportion to vote strength. However, it is also the case in a district-based plurality system. The Westminster system does not choose the 658 best candidates but gives one representative to each geographical district.

Decision rules in legislatures are used to structure a deliberative decision-making process. No legislature proceeds by collecting each legislator's complete preference schedule and then aggregating them into a social ordering. Rather, debates take place, at the end of which votes are taken. In every existing legislature these votes are up-down votes between two alternatives, but it would be theoretically possible to use other procedures. Typically, many of the real decisions are made as a result of negotiation between the various parties or factions. This does not, however, imply that the voting rule is not important. The voting rule determines which coalitions are large enough to carry an outcome. Thus we can think of the voting rule as a rule defining a coalition game between the various parties or factions.[1] Using this framework, it is not necessary

to claim that the procedure produces the objectively "correct" outcome or the "will of the people." To justify such a procedure it is enough to show that it is (1) a fair procedure, satisfying political equality, or (2) a procedure that makes "reasonable" decisions in an environment of limited information about potential alternatives. These criteria are pursued further in chapter 4.

From this point of view, even as an abstraction of what decision ought to be made, the social welfare function is of limited use. From an informational point of view, the demand that legislators have preferences over all possible social states is clearly unrealistic. Indeed, it is almost certainly impossible to enumerate all the possible alternatives. Furthermore, even if we had complete information about all the alternatives, actual legislative negotiation reveals valuable information about intensity of preferences that is not included in the ordinal preference schedules (Buchanan and Tullock 1962). Although this clearly does not overcome the social choice problems outlined by Arrow (see Bernholz 1973; N. Miller 1975, 1977a), it does mean that a social welfare function cannot capture all of the information available to make a social decision. Having said this, the social choice theoretic finding that social decision rules do not give a single determinate outcome, but that instead any alternative can be beaten by some other, will be extremely important, especially in chapter 4.

It is important to distinguish the argument made here from that made by various advocates of "deliberative democracy." Some of these writers (Cohen 1998; Dryzek 2000) have argued that it is possible to distinguish "aggregative" democracy (the type studied by social choice, where preferences are aggregated via a social welfare function) from "deliberative" or "discursive" democracy (where people deliberate and their preferences change). It is argued that if democracy is deliberative, social decision rules are less important, as a consensus may be reached. The problem with this argument (as pointed out by too many authors to list) is that if consensus is not reached, some social decision rule has to be used. In politics decisions typically have to be made, as not making a decision is actually a decision to do nothing (which may well be the decision some people want). Furthermore, the ultimate decision rule will structure the preceding deliberation. If there is a unanimity rule, then people who want (or at least can tolerate) the status quo are under no pressure to compromise and can demand large concessions. If there is majority rule, however, everyone has to look for potential allies. Thus, although the argument here draws heavily on deliberation and negotiation, it does not view these as a substitute for the analysis of social decision rules. Rather, the analysis of social decision rules has to be done in the context of the use of these rules to structure deliberative games.

Let us strictly define what is meant by a seat allocation rule and a social decision rule. Formal definitions are given in the appendix of this chapter. Seat allocation rules map the votes of the population onto an allocation of seats to each alternative (generally a candidate, list, or party), while social decision rules map the votes of the participants (usually legislators) onto a social preference between each pair of legislative alternatives. We can apply the axioms of popular sovereignty and political equality to both these procedures. The application of the axioms, as well as the other qualities we demand of these procedures to render them tractable, will vary for the two types of procedure. The rest of the chapter lays out these qualities.

What We Can Demand of a Democratic Procedure 1: Political Equality

The requirements for popular sovereignty are straightforward. The people must have the final say over any matter, should they choose to exercise it. The concept of political equality requires a bit more work. The conception of equality used here is liberal in that it is specified in terms of individuals and applies to formal political procedures. If we consider individual voters, then the concepts of equality (treating all voters alike), fairness (treating all voters appropriately), and impartiality (not discriminating for inappropriate reasons) are equivalent. Thus we are able to proceed negatively, defining procedures as equal, fair, and impartial if they do not take into account factors that should not be considered. This is the same intuition that underlies Williams's (1962/1971) "relevant reasons" approach to equality and Barry's (1989, 1995) "justice as impartiality." We can then operationalize liberal political equality as the axioms of anonymity (the names of the voters make no difference) and neutrality (the electoral system does not discriminate between alternatives on the basis of their names).

Thus by *liberal political equality* we mean the requirement that procedures treat every individual equally. This may seem the most obvious kind of equality to apply to political institutions, especially in countries with a long tradition of liberal democracy. However, a great deal of the literature on political justice (particularly the electoral systems literature, covered in chapter 3) draws on group conceptions of fairness rather than an individualist conception. Furthermore, various political philosophers, most notably Rawls, have argued that the concept of equality should only be applied to arguments about justice, and not directly to institutions. Finally, some have criticized a liberal conception of equality

for being exclusively formal and not taking account of substantive (and in particular, economic) inequality. I will deal with these objections in turn, before considering how to axiomatize a liberal conception of political equality.

If we consider only individual voters, as opposed to the groups they make up, then the concepts of equality (the demand that individuals be treated the same), fairness (the demand that every individual be treated as he or she ought to be), and impartiality (the demand that how someone is treated should not depend on inappropriate factors, such as personal friendship, class, race, or gender) become equivalent. When we consider the rights of individual citizens in a liberal democratic state, it is extremely hard to escape the conclusion that fairness implies equality. It is difficult for us to argue that one citizen deserves more political rights or better treatment than others. This follows from the standard liberal assumptions (see, for example, Dahl 1988 following Mill 1861/1993 that every individual's interest is worthy of equal consideration and that every individual is the best judge of his or her own interests). However, even if we accept that some individuals are more worthy or more qualified than others, we would still have to find a mutually acceptable procedure to determine *which* citizens are better qualified or more deserving. Given that this is likely to be impossible, equality is the only remaining option that can be described as fair (see Barry 1979). Similarly, if we treat individual voters in different ways, we can hardly argue that we are acting impartially, unless the different treatment is handed out in a strictly random way. It is hard to see how we can justify such discrimination *in terms of individual voters.* Of course, many reasons have been given as to why different voters should be treated differently. However, all these arguments rely on fairness and impartiality in terms of groups and thus are not framed in terms of *liberal* equality.

This equivalence of equality, fairness, and impartiality disappears if we consider groups instead of individual voters. For a start, groups can be of different size. It is not clear that it is fair to treat differently sized groups equally (this certainly violates equal treatment of the individual members of the groups). Neither is it obvious that impartiality is violated if we treat differently sized groups unequally, as to argue this it would be necessary to show that the size of the group is an unacceptable criterion for discriminating. In addition to size differences, some groups may be politically salient, while others may not. Furthermore it is possible to divide a population into groups in many different ways. These different sets of groups may overlap or have cross-cutting membership. Therefore, to make an argument about fairness based on groups, it is necessary first to justify why fairness should be seen in terms of a particular partition of

the voters into groups. This involves both justifying why fairness should be seen in terms of these groups rather than in terms of individuals and justifying why these groups are more appropriate than other possible sets of groups. Having done this, it is necessary to justify a particular conception of fairness between groups.

Two arguments can be made in favor of preferring liberal equality over some form of group-based fairness. First there is the liberal argument that the rights of individuals should enjoy normative priority over the rights of groups that these individuals make up. However, even if we do not accept this doctrinal liberalism, a case can still be made for liberal equality as a fair means to arbitrate between different conceptions of (group-based) fairness. As we have seen there are multiple forms of group-based fairness depending on which set of groups and which principle of allocation we choose. Who has a right to impose their conception of fairness over everyone else? Liberal equality offers a way out, in that it merely demands that all individuals be treated equally. Individual voters can then decide which group identities are salient to them, as opposed to certain forms of group fairness being imposed by the electoral system. We can compare this to the role that political liberalism plays in Rawls (1993/1996). There political liberalism provides a means for arbitrating between competing "comprehensive doctrines" that lay out the values that define the good life. Here liberal equality provides a means of arbitrating between different conceptions of group-based fairness.

So far we have assumed that the concepts of equality, fairness, and impartiality are to be applied directly to political institutions such as electoral systems and legislative rules. However, some theorists deny this and instead take a "constructivist" position, in which equality and fairness are properly applied to a hypothetical decision situation. This hypothetical decision situation then allows the theorist to derive concepts such as fairness to be applied to actual situations. Most famously, Rawls (1971/1999) argues that principles of political justice can be derived by considering what agents would decide in a situation of equality resulting from ignorance of their own circumstances. Institutions are then chosen so as to maximize the probability of legislation that is just in the terms already determined. Beitz (1989) argues even more strongly that concepts such as political equality and fairness should be applied at the level of arguments and not institutions. However, in place of Rawls's original position, Beitz uses the construction from Scanlon (1982) whereby a norm is justified if no person affected by it could reasonably reject it.

However, there are serious problems with arguing that concepts such as equality, fairness, and impartiality should apply to arguments and not institutions. Most significant is the question of who is to determine

what people in a hypothetically constructed decision situation would decide. There is, for example, a large literature arguing that the agents in Rawls's original position would reason quite differently from the way that Rawls argues they would (see Hare 1973/1975 and Harsanyi 1975 for arguments that utilitarianism would be chosen from Rawls's premises). Similarly, as Barry (1995) points out, Scanlon's construction depends heavily on what objections can be considered "reasonable." If different people would come to different conclusions about what would be decided in a hypothetically constructed situation, then we need some means to arbitrate between them. Given that this dispute is to be settled reasonably, without resort to force or authority, we need rules for debate and decision making that are fair. This, however, leads us back to applying the concept of fairness to actual institutions. (This is essentially the argument that Habermas [1995] makes against Rawls [1993/1996], an argument that Rawls [1995/1996] largely accepts.)[2]

Thus there is a strong argument for applying liberal equality directly to institutions. We can accept Barry's (1995) argument that in many cases applying impartiality directly to outcomes leads to ridiculous results, such as the argument that one cannot prefer one's friends over strangers in one's allocation of free time. However, the choice of democratic procedures is precisely the kind of situation where first-level impartiality is appropriate, because it is by these procedures that society determines what is fair and impartial in other circumstances. Fairness requires first-level impartiality because there is no higher court of appeal. (Interestingly, Barry accepts the limitations of theorizing based on a hypothetical construction and argues that these conclusions need to be supplemented by what he refers to as "the empirical method." This involves considering the decisions made in countries where the institutions best approximate the conditions of impartiality. The countries that Barry considers to best meet this criterion are the Scandinavian democracies—that is to say, proportional representation parliamentary systems.)

It could be argued that the emphasis on formal political equality is misguided, as it ignores economic inequalities that can allow certain parts of the political community to dominate others (see Roemer 1999a,b for a recent version of this argument). As Gutmann (1980) argues, it is certainly logically possible for formal and substantive equality to conflict. This creates a situation where we would have to consider the trade-off between the intrinsic value of political equality and the value of substantive justice. We would also have to consider the degree to which formal political equality is meaningful in the presence of severe economic inequality.

However, this trade-off between formal and substantive equality is

highly unlikely to manifest itself in practice. If political equality is violated, the resulting inequality is highly unlikely to benefit the otherwise underprivileged. Underprivileged groups are (almost by definition) unlikely to be powerful enough to get the constitution biased in their favor. If any bias in the democratic procedure is likely to work in favor of the privileged, the best the underprivileged can hope for is political equality. Chapter 8 will show that empirically this is indeed the case: The countries that best approximate our definition of political equality (the small European consensual democracies) are the countries that are economically most egalitarian and redistributive.

We now turn to how the concept of liberal equality can be operationalized. The most efficient way to operationalize liberal equality is to proceed negatively, excluding those considerations whose inclusion would violate equality. This is what we do when we consider equality or fairness in terms of impartiality. It is also the basis of Williams's (1962/ 1971) "relevant reasons" approach to equality—discrimination has to be justified by reasons, and some kinds of reasons cannot be used to morally justify such discrimination. Instead of trying to define what equality is in a specific context, we derive axioms by excluding various considerations.

First, and most obviously, if a social decision rule treats all voters equally, then the outcome should not depend on the names of the voters, or their identities. That is to say, if we permute the names of the voters, this should not change the outcome. Thus the social decision rule needs to satisfy the axiom of *anonymity*. Second, to treat all voters equally, the social decision rule cannot discriminate on the basis of the names or identities of the alternatives. If it did, it would give favorable treatment to the supporters of one alternative over another. Furthermore, such a social decision rule would violate the idea of popular sovereignty. If changing the names of the alternatives changes the result, then the result depends on a built-in bias in the social decision rule, and not solely on the preferences of the voters. Thus political equality also requires the condition of *neutrality*.

Similarly, we can also apply anonymity and neutrality to seat allocation rules. A seat allocation rule is anonymous if changing around the names of the voters does not change the number of seats each alternative receives. A seat allocation rule is neutral if changing the names of the alternatives around does not change the number of seats each receives.

When considering seat allocation rules, the axioms of anonymity and neutrality need to be applied not just to the choice between alternatives, but also to the choice between coalitions of alternatives. This is important because political power is not distributed on the basis of the relative seat share of candidates or parties, but on the relative size of coalitions—or

more precisely in terms of which coalitions have a majority (or superma-jority, if required) of seats and are thus able to elect a government or pass legislation. Applying anonymity to coalitions is straightforward—if the electoral system is anonymous with regard to alternatives, it is automati-cally anonymous with regard to coalitions of alternatives. However, neu-trality to coalitions does not follow automatically from neutrality to alter-natives, and actually represents a considerably stronger requirement. Coalition neutrality implies that the electoral system cannot discriminate between different kinds of coalition. Of course, a case can be made that the electoral system should reward coalitions made up of a small number of alternatives, because this promotes consolidation and stability; or that coalitions of a large number of alternatives should be encouraged be-cause this allows for the representation of a greater number of points of view. The liberal defense of coalition neutrality is that this question should not be settled by the electoral system but by the voters themselves. The electoral system should be neutral between coalitions, because the seat share allocated to each coalition should depend only on the voters, not on a bias built into the system.

What We Can Demand of a Democratic Procedure 2: Other Axiomatic Qualities

Apart from the normatively appealing qualities resulting from political equality, there are other axiomatic qualities that are required for social decision rules to be practically useful. Two of these axioms (decisiveness and nonnegative responsiveness) should be uncontroversial and intuitive. However, the other two (binary independence and transitivity) are logi-cally incompatible. I will argue that for social decision rules the require-ment of binary independence is reasonable, but that we can dispense with the requirement of transitivity. For seat allocation rules, however, a tran-sitive result must occur by definition. However, the requirement of binary independence is unreasonable—indeed it is a quality that any reasonable seat allocation should violate.

For a social decision rule, decisiveness (May 1952) is simply the re-quirement that the rule produce a definite result. An alternative either wins, loses, or draws. In the case of seat allocation rules, every seat has to be allocated or be declared a draw. Note that in most practical political situations, a draw is not an acceptable outcome. If we have a draw in par-liament or in an election, the result typically has to be decided somehow. One way to decide a drawn outcome is at random. This indeed is fre-quently the method of last resort for legislative elections. It is also the

only method that is compatible with political equality. Another means of breaking a draw is to decide the outcome in a nondemocratic manner. For example, a person or committee may be given ex officio power to be the tiebreaker. This, of course, violates anonymity. Alternatively, one alternative may be privileged over the other. This is the case when in the case of a tie the status quo or the incumbent is allowed to remain, violating neutrality.

Nonnegative responsiveness (also known as weak monotonicity) is the requirement that if some voters switch to an alternative and everything else remains equal, it cannot do worse than before. This is a weakening of the quality of positive responsiveness (May 1952). In the case of a social decision rule, positive responsiveness means that if an alternative is winning and some voters switch to it, then it must still win. If it is drawing and some voters switch, then it must win. We can adapt positive responsiveness to seat allocation rules: If two alternatives are receiving the same seat share and some voters switch to an alternative (either at the expense of another alternative or from abstainers), then this alternative must receive a greater seat share than the other alternative.

Positive responsiveness is a strong condition. In terms of social decision rules, it means that draws are knife-edge results: One vote change will turn the draw into a result for one alternative or the other. For seat allocation rules, positive responsiveness implies that if a party wins one extra vote, it acquires a greater seat share, something that is clearly impossible given that seats are not infinitely divisible. Nurmi (1987) suggests nonnegative responsiveness (weak monotonicity) as a more reasonable requirement. In the case of social decision rules, this means that voters' switching to an alternative does not make that alternative do worse (if it was drawing, it must at least still draw). In the case of seat allocation rules, it means that if some voters switch to a party, all other things being equal, then it must at least maintain its seat share. In both cases, extra votes cannot hurt an alternative, but they may sometimes have no effect. Whereas the requirement that even one extra vote must improve the outcome for an alternative is a strong one, the requirement that winning more support does not hurt it is not. We are surely entitled to think of a procedure as perverse if it punishes an alternative for winning more support.

Binary independence and transitivity require more justification, as they are effectively mutually exclusive. Binary independence is essentially the quality that Arrow (1951/1963) referred to as *independence of irrelevant alternatives*. In the context of social decision rules it means that society's choice between alternatives A and B must only depend on the preference of voters between A and B, and not on how voters rate A and

B relative to any third alternative. This can be adapted to seat allocation rules as follows: a seat allocation rule satisfies binary independence if the relative seat share of two alternatives only depends on the preference of voters between these two alternatives. The intuition normally given for binary independence is that if society prefers A to B, it should still prefer A to B if another option becomes available. Furthermore, if this is not so, it is possible to manipulate the outcome by introducing spurious alternatives that do not get chosen but do affect the choice. However, I will justify it on rather different grounds, namely, that it is simply not possible to enumerate what all the alternatives are, so any decision between two alternatives based on a (probably nonrandom) sample of third alternatives will be arbitrary.

Transitivity is the property that if society prefers A over B, and B over C, then it must prefer A over C. (Strictly speaking we can substitute "prefers or is indifferent to" with regard to either A and B or B and C, and still get the result that A is preferred over C.) This allows us to rank all alternatives and have a single alternative (or set of alternatives if they tie) that is preferred to all the others. From the point of view of making a social choice, transitivity clearly makes our life easier. If it was the case that A was preferred to B, B to C, but C was preferred to A, it would not be possible to say which one was preferred by society. Nevertheless, I will argue that this is a property that we must dispense with in social decision rules. In seat allocation rules, however, the property of transitivity is present by definition. Such rules assign a number of seats to each alternative. If A has more seats than B, which in turn has more seats than C, then A must have more seats than C.

Arrow (1951/1963) showed that transitivity and binary independence are logically incompatible unless at least one other quality essential to democratic choice is violated. Specifically, Arrow showed that a social welfare ordering (which is by definition transitive) could not simultaneously satisfy the properties of independence of irrelevant alternatives, universal domain, unanimity (Pareto optimality), and nondictatorship. It is straightforward to show that the latter three qualities are essential to any reasonably democratic decision rule. Universal domain simply means that voters can have any preference ordering. Restricting the preferences that voters are allowed to have clearly violates the most basic democratic principles. Unanimity (Pareto optimality) is the quality that if all voters prefer A over B, then A must be chosen. Violating this principle violates the principle of popular sovereignty. Nondictatorship simply means that a single agent is not always decisive and is obviously necessary for democratic choice.

A common interpretation of Arrow's theorem has been the rather

nihilistic conclusion that there is no nonarbitrary democratic decision rule. Notable among these interpretations are Arrow (1951/1963, 59) himself[3] and Riker's *Liberalism Against Populism* (1982). This reading, however, should be resisted. As Saari (2001) argues, what Arrow's theorem shows is that binary independence and transitivity are mutually incompatible. Arrow's theorem can be seen as a generalization of the famous Condorcet (1788/1995) paradox. Suppose we have three voters making a decision by majority rule, a procedure that satisfies binary independence. Their preferences are as in table 2.1, so that voter 1 prefers candidate a to candidate b to candidate c. Then by majority rule, a beats b, b beats c, but c beats a. What Arrow shows is that any procedure that satisfies binary independence is susceptible to this kind of cycling (strictly speaking, may exhibit intransitivities) or violate one of the other three qualities we have listed as essential to democratic choice. Rather than showing that democratic choice is impossible, Arrow's theorem shows us that we have to make a choice between binary independence and transitivity. We will see that Saari argues for abandoning binary independence, whereas I will argue we need to abandon transitivity in some circumstances.

A possible response to Arrow's result without abandoning transitivity altogether is to weaken this requirement (Sen 1970a). For our purposes, however, this does not help much, as the resulting procedures, although nondictatorial, still seriously violate democratic principles. We can weaken transitivity to quasi transitivity (a beats b, b beats c, implies a beats c, but this is not the case if the relationship between a and b or b and c is one of indifference). This, however, only allows us to replace a dictator with an oligarchy, a single group that is decisive over all social choices (the proof comes from an unpublished paper, Gibbard 1969; see Moulin 1988; Mueller 2003). Alternatively, we could weaken the condition even further to acyclicity (there are no cycles, such that a beats b, b beats c, and c beats a). This dispenses with the need for an oligarchy, but we still have an individual voter who is a veto player if the number of alternatives is greater than the number of voters (Brown 1975; Nakamura 1979). The presence of veto players violates the condition of neutrality,

TABLE 2.1.   The Condorcet Paradox

Voter 1	Voter 2	Voter 3
a	b	c
b	c	a
c	a	b

By majority rule, a beats b, b beats c, and c beats a

in that the status quo can be guaranteed by one voter, but a new alternative cannot. Besides, as we will see in chapter 4, the only binary social decision rule that satisfies the requirement of political equality is majority rule (May 1952), and this clearly is susceptible to cycles. Thus if we want a decision rule that respects political equality, we have to face the choice between binary independence and transitivity.

Binary Independence (Independence of Irrelevant Alternatives)

I will argue that it is appropriate to demand that binary independence be satisfied for social decision rules, but for reasons rather different from those normally given. Arrow (1951/1963, 26–28) justified independence of irrelevant alternatives as a quality we ought to demand of decision rules on the grounds that to do otherwise would be to make the outcome contingent on events that are obviously accidental. If candidate A would win using a certain procedure, it should not make any difference to the outcome if one of the other candidates dies. Arrow criticizes the "rank-order methods frequently used in clubs" on these grounds. Arrow's justification of independence of irrelevant alternatives is similar to Condorcet's (1788/1995, 126) criticism of the Borda procedure.[4]

> But how is it that Paul is not the clear winner when the only difference between himself and Peter is that Peter got thirty-one first places and thirty-nine second, while Paul got thirty-nine first and thirty-one second? Well, out of the thirty-nine voters who put Peter second, ten preferred him to Paul, whereas only one of the thirty-one voters who put Paul second preferred him to Peter. The points method confuses votes comparing Peter and Paul with those comparing either Peter or Paul to James and uses them to judge the relative merits of Peter and Paul. As long as it relies on irrelevant factors to form its judgments, it is bound to lead to error, and that is the real reason why this method is defective for a great many voting patterns, regardless of the particular values assigned to each place. The conventional method is flawed because it ignores elements that should be taken into account and the new one because it takes into account elements which should be ignored.

However, Dummett (1984, 54–59) and Saari (2001) criticize this justification of binary independence on the grounds that comparisons with other alternatives do yield relevant information. Both argue that decisions should not only depend on how many people prefer A to B but also on the intensity with which they prefer. Comparisons to third alternatives

may be an imperfect measure of intensity, but if they are the only information about intensity we have, we should not throw this information away. In response to the argument that a procedure should not be influenced by one candidate withdrawing or dying, Dummett argues that if this happens, we have lost valuable information, so it should not be surprising that our choice may be different. In addition to the fact that limiting ourselves to binary comparisons deprives us of (imperfect) information about intensities, Saari (2001) also argues that it deprives us of transitivity information present in the rankings of the voters, essentially "emasculating" the transitivity requirement of individual voters' preferences.

There are situations where it appears reasonable to use nonbinary comparisons to rank alternatives. Indeed this is commonly used in sports leagues, as Arrow (1951/1963, 27) notes. When deciding whether Arsenal or Manchester United ought to be English Football Champions, we do not simply compare the results of the two games between two clubs. We also compare how the two clubs have fared against Newcastle, Chelsea, Liverpool, and so on. Indeed, if we consider other soccer results, there are many intransitivities (Manchester United beat Charlton, Charlton beats Wolves, but Wolves beats Manchester United). Nevertheless, we have no qualms about transitively ordering clubs at the end of the season using comparisons with third clubs. Where we have a fixed number of candidates for a job, we may ask why it would be more inappropriate to consider nonbinary comparisons.

While a case can be made for using nonbinary comparisons for ranking soccer teams and job candidates, violating binary independence is far more problematic in a legislative setting. The reason for this is not that irrelevant alternatives should not be considered, but that they cannot be considered—at least not all of them. Choosing a government policy is different from choosing a candidate. In an election or job search there are a finite number of candidates. However, all the different packages of policies a legislature could consider cannot even be enumerated. Faced with an infinite (indeed uncountable) number of potential alternatives, any decision based on nonbinary comparisons will depend on what subset of third alternatives we use. But how do we decide what the reference group of alternatives should be? We cannot even take a random sample of alternatives, as we have no way to decide what the sample frame to select from is.[5] The alternatives we are likely to propose will in all probability be highly nonrandom, close to the current status quo. In these circumstances, nonbinary comparisons are likely to be highly arbitrary and thus should be disregarded.

The problem of even describing the set of all possible alternatives in a legislative setting is made more severe by the fact that government pol-

icy is intrinsically multidimensional and the different dimensions of policy affect one another. All government policies that involve spending money affect each other because there is a common budget constraint — if we spend more on health, we must spend less on education or raise taxes. Furthermore, many policy domains affect each other directly — our policy on education affects income distribution, income distribution affects health policy, which in turn affects labor market policies. What an individual or society would choose on each policy dimension depends on what policies are chosen on other dimensions. Deciding policies on different dimensions independently can lead to very poor policies that do not make any sense together, and it may even lead to policy choices that are mutually impossible (for example, deciding to have high public spending, low taxes, and low borrowing). Thus political decisions are more complex than many other kinds of decision. When a group of middle managers decides who to hire, they can take the overall strategic direction of the firm as given, beyond their control. In politics we cannot do this. Popular sovereignty implies that everything is potentially up for grabs.

Using nonbinary comparisons in a situation where alternatives can be made up in an arbitrary manner exposes us to a kind of manipulation different from that we face when choosing a job candidate. We have to worry not only about strategic voting but also about the strategic manipulation of the set of alternatives under consideration. When we consider job candidates, we may vote strategically, misrepresenting our true preferences to try to get a result we prefer, a ploy to which all nondictatorial voting rules are susceptible (Gibbard 1973; Satterthwaite 1975). However, given the costs of applying for the job, we assume that a candidate will not enter the race just to disadvantage another applicant.

Similarly, in a soccer league, the number of teams is fixed. However, in a legislative setting it is always possible to add new alternatives (indeed, to forbid this violates the principle of popular sovereignty). This encourages not just strategic voting but the introduction of spurious alternatives to manipulate the result. If it is advantageous to introduce spurious alternatives, the agenda will soon be overrun by such spurious alternatives proposed by competing factions. The information required to compare all these alternatives becomes immense. Of course, binary decision rules, such as an amendment procedure, can also be manipulated. However, the number of alternatives considered at one time is limited to two. The voters only need the information required to judge between the two alternatives on the table. Furthermore, to have an effect, the new alternative proposed at least has to be relevant in the sense that it can defeat the measure currently under consideration.

Thus our justification of binary independence for social decision

rules does not apply to all situations. We are only concerned with social decision rules used for legislation. Some nonbinary rules have attractive axiomatic qualities (see Young 1974, 1975; Saari 2003 for Borda count; Young 1995 for the Kemeny rule) and may be suitable for some purposes, such as selecting job candidates. However, the number of possible alternatives facing a legislature is not enumerable, and new alternatives can be added in an arbitrary way. In these circumstances, nonbinary rules are either intractable or arbitrary. Thus, binary independence is justified for social decision rules because it provides a tractable basis for legislative bargaining in a world with limited information.

Binary independence is not a quality that we should require of seat allocation rules. Indeed, it is a quality that any reasonable seat allocation rule should violate. Imagine that we have two parties in a legislature, Left and Right, each of which has an equal share of the seats after an election. This means that the ratio of the seat share of the two parties is 1:1. Suppose now that a new party (New Left) enters the legislature. By binary independence, the seat share between Left and Right has to remain 1:1. Thus if New Left receives any seats at all, then the two Left parties between them have a majority. This is clearly unreasonable, as the more left parties enter (or the more existing left parties splinter), the greater the total seat allocation for the left parties. If a new party enters an election, we expect it to win support at the expense of at least one of the existing parties. If New Left enters, but the total vote for the left parties remains constant, it seems reasonable the seat allocation that had gone to Left would be divided between Left and New Left. However, this violates binary independence. Furthermore, in a legislative setting, what matters is not the relative size of parties, but what coalitions they are able to form. In this context, a new party entering the legislature is a potential coalition partner, not an "irrelevant alternative." Binary independence implies that a party can increase its bargaining power by spinning off clones of itself. Therefore any reasonable seat allocation rule will violate binary independence.

Transitivity and Freedom from Cycling

In seat allocation rules as I have defined them transitivity is satisfied automatically. In the case of social decision rules the reasonableness of demanding transitivity (or else a slight relaxation of it such as quasi transitivity or acyclicity) is usually taken for granted. However, I will argue that it can be dispensed with, and (following N. Miller 1983) that cycling is actually normatively appealing. The reason why transitivity is intuitively appealing in social decision rules is that it is assumed as a condi-

tion of individual choice being rational. The assumption here is that so-
cial rationality and individual rationality are similar. One of the starting
points of economic theory is that individuals can rank alternatives in
order of preference. Furthermore, if an individual has cyclical prefer-
ences, preferring good a to b, b to c, and c to a, it is possible for another
individual to keep trading him the more preferred good in the cycle over
and over again, every time making a profit. Another reason why transi-
tivity is desirable in social decision rules is that in a situation of social
choice we have to make a single decision. Transitivity gives us a single
best alternative or at least a set of alternatives that are equally good.

However, the degree to which social decisions should be judged by
the same criteria as individual decisions is open to question. Nicholas
Miller (1983) was the first to suggest that cycling may actually be a desir-
able quality in a social decision rule. His argument grows out of con-
frontation between social choice theory and the empirical study of plu-
ralist democracy. Social choice theory suggests that multidimensional
political competition, as we observe in countries with multiple cross-cut-
ting social cleavages (class, religion, ethnicity, language, region), leads to
cycling and instability. However, the empirical literature shows countries
with such cleavages tend to be politically more stable than those that are
polarized on one dimension. Miller explains this by distinguishing be-
tween policy stability and systemic stability. Cycling may produce insta-
bility in terms of who wins office and what the policy is, although this
instability is more bounded than some previous social choice results sug-
gest. However, this instability increases systemic stability. Because of the
instability associated with cycling, it is always possible for the opposition
to win the next election. Therefore it is in the interest of the opposition
to contest the next election, as opposed to starting a civil war. In the ab-
sence of cycling, there may be permanent winners and permanent losers,
and this unequal stability may lead the losers to reject the political sys-
tem. Therefore in systemic terms, intransitivity is desirable.

Building on this insight, a broader case can be made for the desirabil-
ity of cycling. Consider a legislative bargaining situation where different
parties are negotiating government policy. Cycling implies that any pro-
posal can be beaten by some other proposal. Alternatively it implies that
there are multiple, overlapping winning coalitions. Whatever the current
winning coalition is, and whatever proposal is currently on the table, it
is possible to find an alternative and an alternative coalition to beat it.
That is, it is always possible for those players excluded from the winning
coalition to find some alternative that they can agree on and that will
split the current winning coalition. I will argue that this has some very
appealing normative effects. The fact that there are multiple possible

winning coalitions means that it is unwise to try to treat those excluded from the winning coalition too harshly by infringing on their most vital interests. If the winners do this, the losers will be willing to sell their support to part of the winning coalition at a very low price in exchange for protection of their vital interest, splitting the winning coalition. In chapter 5 I argue that cycling allows us to combine majority rule with minority protection. In chapter 6 I argue that cycling encourages reasonable deliberation. Rather than cycling being a problem for democratic theory, it may be that cycling is what makes democracy as we know it possible.

Furthermore, some of the arguments made in favor of transitive procedures do not really apply in a legislative context. The arguments for transitive, nonbinary procedures are frequently couched in epistemic terms (see, for example, Young 1995). A procedure is good if it identifies that alternative that is most likely to be the best one, given the information we have available. However, in a legislative bargaining situation it is hard to see what we would mean by the "best" option. Rather, the decision is often a distributive one. A transitive social decision rule means that we are able to rank alternatives on a single dimension from best to worst. It is hard to see how this applies here. Certainly some alternatives are worse, in that they are inefficient. However, among those alternatives that are efficient (in the sense that it is not possible to make anyone better off without taking something away from someone else), the difference is distributional and thus multidimensional. It may be possible to rank the various alternatives in accordance with how egalitarian they are, but in practice even this is impossible to do objectively, as different people have different conceptions of what a fair division is. If it is the case that the problem does not have a one-dimensional structure, there is no reason to insist on a transitive social decision rule that reduces the problem to a single dimension. Rather, an intransitive social decision rule that preserves the dimensionality of the problem—the cycling that is present in the collective preferences of the voters—may be a better basis for facilitating reasonable bargaining.

In particular, we should reject the notion that transitivity is necessary for social reason. The fact that we define individual rationality in terms of transitivity does not imply that this axiom must apply to the collective. We should not assume that the collective is just like an individual writ large. Indeed this equation of individual and collective is a result of the same sort of anthropomorphism that social choice theory has largely discredited (most notably Riker's 1982 assault on populism and the notion of the collective will). Instead of defining social reason in terms of a transitive general will (something that is no longer tenable), much of the

political philosophy literature defines it in terms of a communicative process (notably Dewey 1946; Habermas 1984; Rawls 1993/1996). An outcome is reasonable if it is the type of outcome that reasonable people would agree to under reasonable rules. This does not require a transitive decision rule that "discovers" the "correct" solution, but rather a situation in which agents need to persuade one another, producing a reasonable compromise in a situation where there is no right answer.

What We Can Demand of a Democratic Procedure 3:
Protection of Rights, Stability, and Other
Desirable Outcomes

This book develops a theory of democratic institutions in terms of the basic value of political equality. However, no one would argue that this or other procedural qualities are the only things we should value in political institutions. Clearly we are also interested in the kinds of outcomes institutions produce. We are concerned that institutions lead to basic rights being respected and minorities having some means to safeguard their interests. We want institutions to produce reasonable, informed debate, producing reasonable policy decisions and keeping public officials accountable. We also value at least a modicum of stability and are interested in economic growth and equity.

A number of influential theorists have argued that democracy can be justified only in terms of such outcomes, and not in terms of intrinsic qualities. For example, Riker (1982) argues that the results of social choice theory render the idea of democratic choice empty. Schumpeter (1942) and Weber (1978) argue that the public is insufficiently informed or involved for democracy to represent an idea of the popular will (see Warren 1988 on Weber). In all these cases, the only value of democracy is that it allows the periodic removal of elites, which in turn restrains government. This book, of course, rejects the idea that democracy can only be justified in such minimal, instrumental terms, arguing in chapter 4 that democracy can be justified in terms of the fairness of its procedures, and perhaps the reasonable deliberation that results from this.

Nevertheless, it is necessary to consider the performance of democratic procedures in terms of these other values. In particular we need to ask whether there is a trade-off between the value of political equality, on one hand, and the qualities of minority protection, reasonable deliberation, stability, accountability, and economic performance, on the other. Thus chapter 5 considers the relationship between political equality, rights, and minority protection. Chapter 6 considers reasonable deliberation and

accountability. Chapter 8 considers the empirical evidence for the stability and economic performance (both growth and equality) of the countries whose political systems are closest to the ideals of political equality defined here.

Summary

I have outlined the basic concepts and qualities that define democratic choice. Modern democracies involve two sets of procedures. Seat allocation rules translate the electorate's votes into allocations of seats in decision-making bodies. Social decision rules are used by these bodies to compare policy alternatives and prospective governments. It is important that we not consider democracy a single procedure (a social choice function, in the language of social choice theory) that directly maps people's preferences onto outcomes. Rather, political institutions consist of separate sets of procedures that structure games in which people compete to be representatives, on one hand, and they negotiate and deliberate, on the other.

Following Dahl (1956) we start with the basic assumption that democracy can be defined in terms of popular sovereignty and political equality. I have defined political equality in a liberal sense, in that it is based on procedures treating individual voters equally. This individualism can be defended either in terms of doctrinal liberalism (the individual takes normative precedence over groups individuals make up) or on the grounds that it provides the only neutral way to arbitrate between the claims of different groups. The emphasis on procedure was justified on the grounds that politically fair procedures are a sine qua non of democracy, necessary for any of the other attributes we may consider democratically desirable. It is possible to operationalize this liberal conception of political equality as defined in terms of anonymity (all voters are treated alike) and neutrality (all candidates or alternatives are treated alike). These qualities can be defined to apply to both seat allocation rules and social decision rules.

There are other axiomatic qualities that reasonable democratic procedures must have. Decisiveness and nonnegative responsiveness should be uncontroversial. However, binary independence and transitivity are logically incompatible in a democratic procedure, as demonstrated by Arrow (1951/1963). Which axiom we should choose depends on the type of procedure. I have argued that we should demand that a social decision rule satisfy binary independence, but for a reason quite different from that given by Arrow and most of the social choice literature. Binary in-

dependence is necessary because the set of possible government policies cannot be enumerated, which makes binary decision rules necessary for the sake of tractability. Furthermore, political decision making involves a bargaining process, to which binary procedures are well suited. However, I have argued that transitivity is not essential in a social decision rule and that cycling may actually be a good thing, protecting minorities from permanent majorities and preserving the possibility that today's losers could be tomorrow's winners. With seat allocation rules, however, this situation is reversed. Binary independence is a positively undesirable quality in a seat allocation rule, while transitivity is satisfied by definition.

In addition to the qualities intrinsic to democratic procedures, such as political equality, we are also interested in the instrumental effects of democratic institutions. That is, we are interested in whether democracy leads to outcomes like the protection of minorities, accountability of governments, rational decision making, and at least a modicum of stability. It has been argued that democracy only has value in terms of such instrumental benefits, with democratic procedures having no intrinsic value (Schumpeter 1942; Riker 1982). While this book rejects this argument in chapter 4, the instrumental effects of democratic institutions are considered in chapters 5, 6, and 8.

APPENDIX: FORMAL DEFINITIONS OF AXIOMS

SEAT ALLOCATION RULES

Let us define the set of eligible voters as N, with voters numbered $1 \ldots n$, and the set of alternatives A, numbered $1 \ldots a$. The voting correspondence V is defined over the Cartesian product $N \times A$, with $_{i \in N}V_{j \in A}$ reading "i votes for alternative j." Assigning the value 1 for true and 0 for false, $\forall\, i \in N \sum_{j \in A}{}_i V_j \leq 1$. (Each individual either votes for one alternative or does not vote.) The function T maps the voting correspondence into the total vote for each alternative: $T : V \rightarrow [0,n]^A$. The function E maps the voting correspondence into the seat share for each alternative: $E : V \rightarrow [0,1]^A$. We will assume that seats are infinitely divisible, to abstract from rounding problems.

We can define the following properties of the seat share function E.

Anonymity: Let σ be a function that permutes N. Then E is anonymous if $E(V) = E(\sigma V)$.

Neutrality: Let π be a function that permutes A. Then E is neutral if $\pi E(V) = E(\pi V)$.

Decisiveness: E is decisive iff $\forall\, V, \forall\, i \in A : E_i(V) \in [0,1]$. This is automatically satisfied by the definition of E.

Nonnegative (positive) responsiveness: Let V' be a vote pattern over $N \times A$. Let $V'' = V'$, except that some voters or abstainers have switched to alternative j:

$$(_iV'_j \Rightarrow _iV''_j; \exists\, i \in N : _iV''_j \wedge \sim_iV'_j; \forall\, i \in N : \sim_iV''_j, \forall\, k \in A, _iV'_k \Leftrightarrow _iV''_k).$$

Function E is nonnegatively responsive iff

$$\forall\, (j, k \in A : k \neq j)\ E_j(V') = E_i(V') \Rightarrow E_j(V'') \geq E_i(V'').$$

Function E is positively responsive iff $\forall\, (j, k \in A : k \neq j)\ E_j(V') = E_i(V') \Rightarrow E_j(V'') > E_i(V'')$.

Transitivity: E is transitive iff

$$((E_{i \in A} \geq E_{j \in A}) \wedge (E_{j \in A} \geq E_{k \in A}) \Rightarrow E_{i \in A} \geq E_{k \in A})$$

$$\wedge\, ((E_{i \in A} \geq E_{j \in A}) \wedge (E_{j \in A} \geq E_{k \in A}) \wedge ((E_{i \in A} > E_{j \in A})$$

$$\vee\, (E_{j \in A} > E_{k \in A})) \Rightarrow E_{i \in A} > E_{k \in A}).$$

This is automatically satisfied by the definition of E.

Binary independence: E satisfies binary independence iff $\forall\, A', A'' : i, j \in A'$, $A'' : E_{i \in A'}/E_{j \in A'} = E_{i \in A''}/E_{j \in A''}$. The text argues that binary independence is not a property that a seat allocation rule should satisfy.

SOCIAL DECISION RULES

Let us define the set of eligible voters as N, with voters numbered $1 \ldots n$, and the set of alternatives A, numbered $1 \ldots a$. The preference correspondence R is defined over the Cartesian product $N \times A \times A$, with $_{b \in A}R_{ic \in A}$ reading "i prefers alternative b to c or is indifferent between them." For convenience let us define the strict preference relation S, where $_{b \in A}S_{ic \in A}$ reads "i strictly prefers b to c" and $_{b \in A}S_{ic \in A} \Leftrightarrow (_{b \in A}R_{ic \in A}) \wedge \sim(_{c \in A}R_{ib \in A})$.

Let us assume that R is transitive so that

$$((_{b \in A}R_{ic \in A}) \wedge (_{c \in A}R_{id \in A}) \Rightarrow _{b \in A}R_{id \in A})$$

$$\wedge\, ((_{b \in A}R_{ic \in A}) \wedge (_{c \in A}R_{id \in A}) \wedge ((_{b \in A}S_{ic \in A}) \vee (_{c \in A}S_{id \in A})) \Rightarrow _{b \in A}S_{id \in A}).$$

Let us define the social preference correspondence R_S over the Cartesian product $A \times A$, with $_{b \in A}R_{Sc \in A}$ reading "society prefers alternative b to c or is indifferent between them." Let S_S be the related strict preference relation. Let the social preference function U map individual preferences onto social preferences: $U : R \rightarrow R_S$.

Decisiveness: R_S is decisive iff $\forall\, b, c \in A : {}_{b \in A}R_{Sc \in A} \vee {}_{c \in A}R_{Sb \in A}$.

Nonnegative (positive) responsiveness: Let U', U'' be preference profiles over $N \times A \times A$. Let U'' be identical to U', except that at least one person has either switched from preferring b to preferring c, has switched from indifference to preferring c, or has switched from preferring b to indifference. Formally:

$(\forall\,i \in N : {}_cS_i(U')_b \Rightarrow {}_cS_i(U'')_b;\ {}_cR_i(U')_b \Rightarrow {}_cR_i(U'')_b)$

$\quad \wedge\, (\exists\,i \in N : ({}_bS_i(U')_c \wedge {}_cR_i(U'')_b) \vee ({}_bR_i(U')_c \wedge {}_cS_i(U'')_b))$

$\quad \wedge\, (\forall\,i \in N, \forall\,d, e \in A_{/b,c} : {}_dR_i(U')_e \Leftrightarrow {}_dR_i(U'')_e).$

Correspondence R^S is positively responsive iff ${}_bR_i(U')_c \Rightarrow {}_bS_i(U'')_c$, and R^S is non-negatively responsive iff $({}_bR_i(U')_c \Rightarrow {}_bR_i(U'')_c) \wedge ({}_bS_i(U')_c \Rightarrow {}_bS_i(U'')_c)$.

Transitivity: R_S is transitive iff

$(({}_{b\in A}R_{Sc\in A}) \wedge ({}_{c\in A}R_{Sd\in A}) \Rightarrow {}_{b\in A}R_{Sd\in A})$

$\quad \wedge\, (({}_{b\in A}R_{Sc\in A}) \wedge ({}_{c\in A}R_{Sd\in A}) \wedge (({}_{b\in A}S_{Sc\in A}) \vee ({}_{c\in A}S_{Sd\in A})) \Rightarrow {}_{b\in A}S_{Sd\in A}).$

Binary independence: Let R_S^A be the social preference relation defined over set of alternatives A. Relation R_S is binary independent iff

$\forall\,A', A'' : b, c \in A', A'' : {}_{b\in A'}R_{Sc\in A'}^{A'} \Leftrightarrow {}_{b\in A''}R_{Sc\in A''}^{A''},$

where A' and A'' are two sets of alternatives.