POWER LAWS, SCALING, AND FRACTALS IN THE MOST LETHAL INTERNATIONAL AND CIVIL WARS

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The most lethal international and civil wars in modern history (1816–present) have caused tens of millions of fatalities ($\sim 10^7$) measured in battle deaths alone. The even more catastrophic loss of human life in terms of total casualties and war-related civilian deaths caused by these interstate and domestic conflicts combined during the past two centuries has been even greater (perhaps $\sim 10^8$, in the hundreds of millions range). In spite of their theoretical and policy significance (Clemens and Singer 2000), an in-depth analysis of the set of highest-magnitude international and civil wars has never been conducted, although several decades have passed since the Correlates of War Project has been reporting extensive systematic data and numerous findings on other types of wars (Singer and Small 1972; Small and Singer 1982; Vasquez 2000).

In this study we use complexity theory to analyze and compare the so-called power law behavior of the highest-magnitude international wars and civil wars along dimensions of onset (time between onsets), fatalities (battle deaths or intensity) and duration, testing specific hypotheses and quantitative models that account for their occurrence. To clarify matters at the outset, the term *power law* has no connection of any kind with the conventional usage of the term *power* in political science. Instead, it is a verbal description of a mathematical function describing the uniform decline in values according to a numerical power (2, 3, . . .). Small and Singer (1982) and subsequent studies (summarized in Geller and Singer 1998) report numerous analyses of war, but power law analyses focused on this specific class of most lethal wars have never before been conducted. Among other findings, we demonstrate that the most lethal international and civil wars obey a uniform class of power laws with respect to onset, fatalities, and duration. The power law, therefore, is a description of conflict behavior intrinsic to the conflict process itself.

These new results, based solely on Correlates of War Project data produced by J. David Singer and his collaborators, are significant for several reasons: (1) they provide the first solid replication of Lewis F. Richardson's (1948, 1960) original discovery of the power law behavior of war magnitude, which until now had been based exclusively on Richardson's much older "deadly quarrels" data; and (2) they extend the power law behavior of warfare to other theoretically important spatiotemporal dimensions of warfare, such as time-between-onsets and duration, not just the single magnitude dimension tested by Richardson. Thus, the power law pattern of warfare is now shown to govern not just one (magnitude), but a minimum of three spatiotemporal dimensions of warfare: time of onset, magnitude, and duration. In turn, this finding is significant because the so-called scaling property of these highly lethal wars, associated with their power law behavior, reveals previously unknown fractal properties that have implications for theoretical research as well as for early warning and mitigation policies. Inter alia, our findings account for the "long peace" phenomenon, which we demonstrate is infrequent but certain, given a sufficiently long historical epoch. As we discuss in this chapter, the multidimensional scaling of high-magnitude warfare according to uniform power laws may also indicate that the international system produces these highly lethal events as a result of "self-organized criticality" (Bak 1996), a previously undiscovered phenomenon in international relations. The high scientific reliability and validity of the modern COW data sets available today make these and other significant inferences possible, by combining the precision of systematic empirical observation with the power of estimated formal models.

This chapter contains five sections. The first provides theoretical and empirical background on power laws, explaining what they are and how complexity theory provides some insightful conceptual, modeling, and empirical tools for advancing our understanding of warfare. The purpose here is not to provide a primer on complexity theory (Badii and Politi 1997; Bak 1996; Meakin 1998; Richards 2000; Schroeder 1991; Waldrop 1995) but rather to highlight the theoretical implications of empirical power laws that are observed in distributions of data. The second section explains the methods used in this study. Our findings are reported in the third section, followed by a discussion of findings. The last section provides a summary.

BACKGROUND

Power Law Behavior and Complexity Concepts

What is a power law? Informally, a *power law* describes a variable X that has *many* (a high frequency of) *small* values, *some mid-range* values, and only a *few large* values, as opposed to the opposite (many large and few small) or some other pattern (Cioffi-Revilla 2003). By contrast, a "normal" (Gaussian) variable has a distribution with many midrange values and few extreme values at both high and low ends; a "uniform" variable has a distribution with the same number of values across the entire range. Therefore, a power law is characterized by the unique "many-some-few" pattern of symmetry (Schroeder 1991). In the social sciences, power laws were first discovered in areas such as linguistics (Zipf 1949), economics (Pareto 1927), sociology (Simon 1957), conflict analysis (Richardson 1941; see also Midlarsky 1989), and geography (Berry and Pred 1965). However, it was not until the recent formulation of *complexity theory* (Badii and Politi 1997; Bak 1996; Schroeder 1991; Waldrop 1995) that power laws acquired increased theoretical relevance for the insights they provide into the underlying (latent) causal dynamic mechanisms that produce the unique or "signature" pattern of "many-some-few" frequencies.

More rigorously, a power law distribution is a nonlinear mathematical model from complexity theory that specifies that the frequencies associated with values of a given variable X are distributed according to an *inverse function,* such that increasing values of X occur with decreasing frequency. Formally,

$$N_c = a'/10^{bX}, \tag{1}$$

where N_c is the *cumulative frequency* of values of X, and a' and b are constants that determine the *range* of values and the *scaling* proportion for $x \in X$, respectively. The nonlinear form of equation (1), or hyperbolic distribution, is *linearized* by taking common logarithms on both sides and rearranging terms, yielding

$$\log N_c = a - b\,X, \tag{2}$$

where $a = \log a'$. The graphs of equations (1) and (2) are shown in figure 1. (Throughout this chapter, "log" denotes $\log_{10}$.) Note that whereas the original power law, equation (1), is *nonlinear* (fig. 1a), the transformed power law, equation (2), is *linear* (fig. 1b).

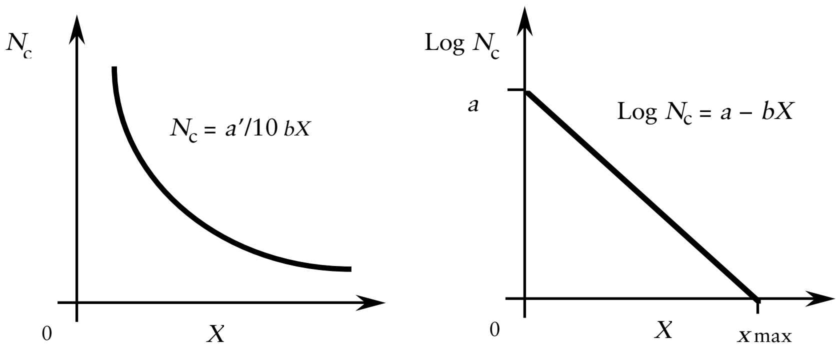

Fig. 1. Graphs of a power law

A power law, or a given variable *X* obeying a hyperbolic distribution, has the following distinctive properties associated with *complex systems* that are governed by underlying *nonlinear dynamics:* (1) self-similarity, (2) scaling, (3) fractal dimension, (4) criticality and underlying driven threshold systems, and (5) long-range interactions.

Self-similarity. When *X* obeys a power law, a recurring pattern of *constant proportion* occurs across the entire range of values of *X*, as highlighted by the linear graph in figure 1b. The graph of the frequency function is as linear in the low range of values as it is in the high range, and everywhere in between. This type of global symmetry is known as *self-similarity* in complexity theory. Self-similarity is also said to be an *emergent* property because it applies to the whole set of values, to an entire distribution of observations, not to individual values or elements.

Scaling. The property of self-similarity is also known as *scaling.* Lewis F. Richardson (1948, 1960) discovered that “deadly quarrels” scale with respect to *magnitude* μ (see also Midlarsky 1989). Do the highest-magnitude wars measured by the Correlates of War Project also scale? (Note that “deadly quarrels” and “COW wars” constitute different sets of sample points, so there is no a priori guarantee that they both scale.) More generally, do other dimensions besides μ, such as *time of onset* and conflict *duration,* also scale? Do different *types* of warfare (interstate, extrasystemic, international, civil, combined) also scale, or scale differently? Note that scaling occurs if and only if a variable obeys a power law. (Most biological organisms do *not* scale.) Is it possible for scaling to occur in the behavior of highest-magnitude warfare, in spite of major changes in technology, population patterns, international system composition, and other arguably significant changes that have occurred during the past two centuries? Intuition would say “no.”

Fractal dimension. If the slope b in equations (1) and (2) were allowed to assume only integer values (1, 2, 3, 4, . . .) then the frequencies associated with each value would decrease inversely by the power of such integer proportions, as in equation (1). However, when b assumes fractional values then the range of proportions is itself continuous, no longer discrete as in Euclidean space. Thus, the b-value in a power law is called the *fractal dimension* (Mandelbrot 1977; Meakin 1998). Note that scaling vanishes as the slope decreases ($b \to 0$), because all values of X assume the same frequency when $b = 0$, so from a scaling perspective a uniform random variable exists in a 0-dimensional space. A hyperbolic power law ($b = 1$) yields a 1-dimensional space. A quadratic power law ($b = 2$) yields a 2-dimensional space. In general, a b-power distribution yields a b-dimensional space, and fractional values of b yield fractal dimensions embedded within Euclidean space. Thus, for $0 < b < 1$ (as we will demonstrate for warfare) the fractal dimensionality is between a point and a line; for $1 < b < 2$ it is between a line and a plane; for $2 < b < 3$ it is between a plane and something else.

Criticality and underlying driven threshold systems. Scaling phenomena are produced by an underlying system that is *driven* by slowly evolving input processes to a phase of criticality (Rundle et al. 1996, 2000). Although the input driving the system can behave continuously, the state variables can change abruptly inside what is called a critical *bifurcation* region, producing scaled phenomena. Is the international system a "driven threshold" system in the sense of complexity theory (Cioffi-Revilla and Rundle 1999, 2000)? The demonstration of extensive scaling for multiple dimensions of warfare—such as time between onsets, magnitude, and duration—would provide significant support for such a conjecture. As discussed later, in the case of the international system the driving dynamics can be interpreted as slowly evolving changes in national attributes (for example, military budgets and capabilities), power distributions, or technological changes, which are known to affect decision-making calculations on war and peace.

Long-range interactions. Scaling phenomena are produced by systems that evolve into a critical phase where *long-range interactions* occur. A system governed by only nearest-neighbor or *local* interactions will tend to produce mostly normally distributed phenomena, not power law phenomena with significant left-skewness (long or "thick" right tail). Long-range interactions involving alliances, remote force deployments, power projection, and significant loss-of-power gradients are well-documented for many of the highest-magnitude wars in the international system. Conversely, long-range interactions are rare for lower-magnitude wars. This could explain why most "world wars" are also "global wars," and vice versa. However, most wars among neighbors (short-range interactions) are neither world wars nor global wars.

The preceding concepts from complexity theory are all related to power laws, such that when a power law behavior is observed in a given empirical domain—such as warfare—these ideas may suggest new insights on the phenomenon under investigation. Thus, power laws can be interpreted as diagnostic indicators of self-similarity, scaling, fractals, criticality, driven threshold dynamics, long-range interactions, and other complex phenomena.

Power Laws of Warfare

Given the preceding concepts, in this study we examine the power law behavior of highest-magnitude warfare with respect to three separate (putatively independent) dimensions of warfare:

- Time between consecutive onsets T
- Richardson magnitude μ
- Duration D

As detailed later, our hypothesis is that all three of these key spatiotemporal dimensions of war—not just Richardson's magnitude—obey uniform power laws, that is, equations (1) and (2). If so, then the preceding insights and implications from complexity theory—the properties of self-similarity, scaling, fractal dimension, criticality, driven threshold systems, and long-range interactions—become relevant for better understanding high-magnitude warfare. The general idea is analogous to that which occurs when exponential behavior is observed in the aggregate behavior of a given population; additional insights can be derived from the exponential laws to advance one's understanding of the population's behavior. Conversely, if power laws do *not* govern key dimensions of high-magnitude warfare such as onset time, magnitude, and duration, then these ideas become less relevant for understanding these conflicts, and a different set of concepts should be developed.

METHOD

The previous section provided theoretical motivation and foundations for modeling high-magnitude warfare with power law models. In this section we explain the data, methods, and standards of inference used in this study.

Data

The data sets used in this study were obtained from the three main original warfare files of the Correlates of War Project:

1. the Inter-State War Data, 1816–1997, version 3.0, taken from http://pss.la.psu.edu/ISWarFormat.htm;
2. the Extra-State War Data, 1816–1997, version 3.0, taken from http://pss.la.psu.edu/ESWarFormat.htm; and
3. the Civil War Data, 1816–1980 ($N = 106$), taken from Small and Singer (1982).

These are the standard war files of the Correlates of War Project; the same ones that are used by most of the chapters in this book. The availability of these data sets through the Internet marks a significant scientific improvement with respect to earlier modes of dissemination. Accordingly, each of the analyses conducted in our study can be replicated with the same data downloaded from these URLs.

In comparative terms, the earlier Richardson discovery of the power law of war magnitude μ was based on his earlier "deadly quarrels" data set, which would have been a sample roughly equivalent to the sum total (union) of all three of the modern COW data sets. Hence, this is a more focused and targeted analysis aimed at both (1) replicating Richardson and (2) extending the domain of power laws to temporal dimensions (onset and duration) and the separate and specific set of high-magnitude wars (international and civil wars).

Variables

For each war sample (international, civil, and combined) we used the following variables: onset year τ, fatalities F, and duration D. In turn, based on the COW-defined variables (Small and Singer 1982; Geller and Singer 1998) we derived the following additional variables: (1) *time between onsets* T, defined as

$$T = \tau_{i+1} - \tau_i,$$

where $i = 1, 2, 3, \ldots, N$; (2) *Richardson magnitude* μ, defined as

$$\mu = \log F; \text{ and}$$

(3) *war duration* D, defined as the length of time a war lasts.

In the statistical analyses reported in the next section we used the Richardson magnitude μ and not F, because the latter ranges across several orders of magnitude, so it is more appropriate to use the logarithmic scale of μ rather than values of F to test for a given power law. In addition, as noted by Richardson (1960, 6), values of μ are less susceptible to measurement error than values of F.

Hypotheses

Our general research hypothesis is that each of the three basic dimensions of warfare (T, μ, and D) conforms to a power law with constant a and fractal slope b, as in equations (1) and (2). Accordingly, our specific research hypotheses were formulated as follows.

$$H_1: \quad \log N_c(T) = a_1 - b_1\, T, \tag{3}$$

$$H_2: \quad \log N_c(\mu) = a_2 - b_2\, \mu, \tag{4}$$

$$H_3: \quad \log N_c(D) = a_3 - b_3\, D. \tag{5}$$

The corresponding null hypothesis H_0 for a given dimension X was that X does not follow a power law. Empirically, this would mean that a poor fit would result between the ranked-log frequency data and the linearized power law (eqs. 3–5).

Analysis

The power law analysis conducted in this study aimed at replicating and extending earlier analyses of the scaling properties of warfare dimensions (Richardson 1948, 1960; Cioffi-Revilla 2000b) to the specific class of highest-magnitude wars. The power law analysis consisted of testing equations (3)–(5) on the three sets of COW Project data (international wars, civil wars, and all wars combined). The standard procedure for testing the power law behavior of a variable X with a set of values $x_1, x_2, x_3, \ldots, N = \{x_i\}$, consists of (1) ranking the values of X to obtain a ranked set of values $\langle x_i \rangle \in X$; (2) calculating the cumulative frequency $N_c = \Sigma_i f_i$ for increasing values of X, where f_i is the frequency of the ith ordered value; and (3) regressing the log N_c values against values of X. Examples of this basic procedure may be found in Axtell (1999), Barabási and Albert (1999), Nishenko and Barton (1996), Richardson (1960, 149), Weiss (1963), and Wyss and Wiemer (2000, 1337).

All statistical calculations were performed with Statistica™ version 4.1 for Macintosh (see www.statsoft.com) running Mac OS 9.1.

Inference

For purposes of establishing valid inferences, we used standard goodness-of-fit criteria for linear models, given that equations (3)–(5) are rendered in linear form: t-ratios of a and b estimates, coefficient of determination R^2, the F-ratio, and significance levels of the preceding statistics. By convention, the .05 level of significance is taken as sufficient, with lower levels indicative of increasingly high significance. Surprisingly, much of the extant literature relies solely on the R^2 value, which provides a weak or ambiguous standard when used as the sole criterion (King 1986).

We also compared results derived from the empirical data sets with nonscaling results obtained from a simulated (synthetic) set of independent and identically distributed (i.i.d.) uniform random variables U_i. As we demonstrate in the next section, a uniform random variable (r.v.) yields a set of baseline estimates that facilitate the interpretation of results derived from real data. A uniform r.v. U has the following distinguishing properties, which are different from a power law:

1. Every value $u \in U$ is equiprobable (i.e., a low frequency of high values, or a high frequency of low values, is not possible);
2. No scaling occurs (the c.d.f. $G(x)$ is monotonic, $d^2G/dx^2 = 0$); and
3. The fractal slope is equal to zero ($b = 0$). Formally, $b \rightarrow 0$ as $p(x) \rightarrow p(u)$, where $p(\cdot)$ is the p.d.f. for the r.v. X and the r.v. U, respectively.

In particular, the occurrence of the third property in empirical data is a sufficient condition for rejecting the research hypothesis (H_r: warfare dimension X scales with slope b, where $X = T$, μ, or D) and accepting the null hypotheses (H_0: $X = U$), regardless of the associated R^2 value. Conversely, we accept the research hypothesis that X scales with slope b whenever $b \neq 0$, with high t-ratio, and the F-ratio is significant at $p < .05$.

RESULTS

First, we examine results from the total set of all wars consisting of the combination of the interstate, extrastate, and civil war data (Cioffi-Revilla 2000b). These findings are presented in figure 2, which shows the power law plots for the time between onsets (τ), Richardson magnitude (μ), and duration (*D*) for wars of all magnitude (low and high). As shown in figure 2, some of the curves exhibit a less than perfect fit. Specifically, that segment of the plot containing the largest wars (i.e., highest magnitude range) does not appear to conform to the pattern exhibited by the remainder. This is especially true for fatalities and duration. This upper-range "bending" of the data at the bottom of these plots requires explanation and a separate analysis to determine if indeed these large wars conform to a power law with somewhat different parameters or to some other as yet unspecified pattern.

"Highest-magnitude wars" are operationally defined as those wars that rank within the upper decile (top 10 percent) of the distribution of fatalities, as shown in table 1. The cumulative number of fatalities produced by these twenty-four wars alone, the highest-magnitude outbreaks in the international system since 1816, totals approximately

TABLE 1. Largest International and Civil Wars Ranked by Fatalities (intensity), 1816–Present

COW No.	War Name	Onset Year τ	Duration D (days)	Fatalities F	Richardson Magnitude $\mu = \log F$
139	World War II	1939	2,175	16,634,907	7.22
106	World War I	1914	1,567	8,578,031	6.93
652	China CW	1860	1,650	2,000,025	6.30
199	Iran-Iraq	1980	2,890	1,250,000	6.10
163	Vietnamese	1965	3,735	1,021,442	6.01
868	Nigeria CW	1967	906	1,000,000	6.00
784	China CW	1946	1,476	1,000,000	6.00
130	Sino-Japanese	1937	1,615	1,000,000	6.00
151	Korean	1950	1,130	909,833	5.96
778	Spain CW	1936	972	658,300	5.82
658	USA CW	1861	1,440	650,000	5.81
421	Fr.-Indochinese	1945	3,105	600,000	5.78
745	Russia CW	1917	1,026	502,225	5.70
880	Pakistan CW	1971	249	500,000	5.70
049	Lopez	1864	1,936	310,000	5.49
835	Vietnam CW	1960	1,836	302,000	5.48
381	Spanish-Cuban	1895	1,152	300,000	5.48
317	Franco-Algerian	1839	2,975	300,000	5.48
802	Columbia CW	1949	4,788	300,000	5.48
061	Russo-Turkish	1877	267	285,000	5.46
022	Crimean	1853	861	264,200	5.42
853	Sudan CW	1963	3,027	250,000	5.40
727	Mexico CW	1910	3,285	250,000	5.40
058	Franco-Prussian	1870	223	204,313	5.31

Source: Correlates of War Project, files cited in the Methods: Data section.
Note: CW = civil war. $N = 24$ wars.

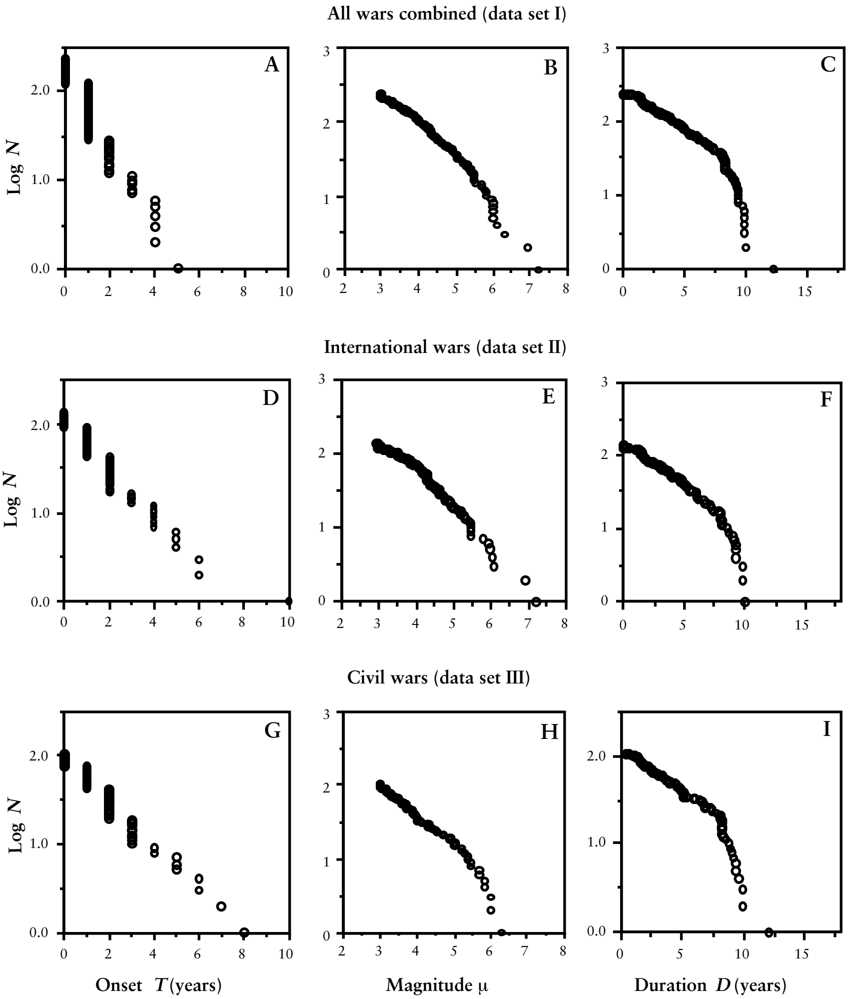

Fig. 2. Power law plots of onset T, magnitude μ, and duration of warfare based on Correlates of War Project data. (Adapted from Cioffi-Revilla 2000a.)

39,070,276 or $\approx 4 \times 10^7$ fatalities. With few exceptions, this class of high-intensity wars corresponds to those recently highlighted by Clemens and Singer (2000). As a class, these highest-magnitude cases account for the overwhelming majority of loss of human life by organized violence in modern history.

Do the high-magnitude wars shown in table 1 obey power laws with respect to the three basic dimensions of onset, magnitude, and duration? This is an important puzzle to address, given the devastating nature of these wars and the special properties of power laws and related behavior. Note that while some of these dimensions of warfare have been studied from a stochastic perspective (extensive references found in Cioffi-Revilla 1998, 52–53), only the magnitude μ variable has been analyzed for power law behavior—the others have not. Lewis F. Richardson (1941, 1960) was the first to discover the power law behavior of warfare magnitude , based on his data set of "deadly quarrels." Surprisingly, no one in the past fifty years has investigated the power law behavior of warfare using data from the Correlates of War Project, nor has a focused study been conducted on warfare in the high-magnitude range.

With respect to the wars in table 1, a power law model of such wars would capture the pattern (first discovered by Richardson for "deadly quarrels") that there have been very few wars as intense as World War II, but there have been many wars with lower magnitude. In fact, table 1 shows that since 1816 there has been only one large war at magnitude 7, only seven wars at magnitude 6, and many more at magnitude 5. This is precisely the power law pattern, which is neither "normal" nor "uniform."

The purpose of this study is to analyze and compare the power law behavior present (or absent, as the case may be) in the occurrence of these highest-magnitude wars, as measured by the Correlates of War Project. Given the implications of power law behavior, the puzzling pattern of highest-magnitude warfare is of fundamental and enduring scientific interest.

Based on table 1, we used the following three data sets in this study.

Data set I. *Most lethal international wars* (N = 13), produced by combining interstate wars and extrastate wars, ranking them by fatalities, and taking those cases in the top decile of the distribution;

Data set II. *Most lethal civil wars* (N = 11), produced by ranking all the civil war cases by fatalities, and taking those cases in the top decile of the distribution; and

Data set III. *Most lethal wars* (N = 24), combining data sets I and II), produced by merging the largest international wars with the largest civil wars, rank ordering them by fatalities, and taking those cases in the top decile of the distribution.

Note that the war cases included in our third data set (see table 1), containing international wars and civil wars combined, most closely resembles Richardson's (1960; see also Wilkinson 1980) pioneering data set of "deadly quarrels"—but only the top decile of cases when ranked by magnitude. Our third data set also resembles, both in content and size, the recent Clemens and Singer combined sample of international wars and civil wars (2000).

Table 2 and figures 3 (a–c), 4, and 5 show the parameter estimates

TABLE 2. Scaling Parameter Estimates (*a, b*) for Power Laws of Onset *T*, Magnitude μ, and Duration *D* Dimensions of Largest-Scale Warfare, 1816–Present

Warfare Dimension *X*	Intercept *a*	Fractal Slope *b*	*N*	R^2	*F*	Power Law Plot
I. Largest International Wars						
T	1.28	−0.05	12	0.91	96.48	Figure 3A
	(19.86)	(9.82)				
μ	3.98	−0.55	13	0.95	202.17	Figure 3B
	(17.46)	(14.22)				
D	1.27	−0.10	13	0.91	115.86	Figure 3C
	(22.75)	(10.76)				
II. Largest Civil Wars						
T	0.88	−0.02	10	0.85	45.11	Figure 4A
	(16.51)	(6.72)				
μ	6.94	−1.09	11	0.95	165.54	Figure 4B
	(14.27)	(12.87)				
D	1.25	−0.10	11	0.98	608.69	Figure 4C
	(48.53)	(24.67)				
III. Largest Wars (international and civil combined)						
T	1.37	−0.06	23	0.97	736.05	Figure 5A
	(70.93)	(27.13)				
μ	5.31	−0.74	24	0.95	448.11	Figure 5B
	(25.94)	(21.17)				
D	1.54	−0.11	24	0.96	486.23	Figure 5C
	(52.33)	(22.05)				
IV. Uniform Random Data (Monte Carlo simulation)						
U	3.20	0.00	1,000	0.77	3,317.7	Figure 6
	(250.19)	(57.60)				
V. Richardson's Deadly Quarrels						
μ	4.13	−0.54	5	0.997	874.61	Richardson
	(43.27)	(29.57)				(1960, 149, fig. 4)

Source: Calculated by the authors.

Note: *t*-ratios of estimates are given in parentheses.

All estimates of *a* and *b*, as well as values of R^2 and *F,* are significant, $p < .01$. Most estimates are highly significant, $p < .001$, as seen from the high *t*-ratios given in parentheses below the values of *a* and *b*.

$N = 5$ for Richardson's deadly quarrels because of aggregation.

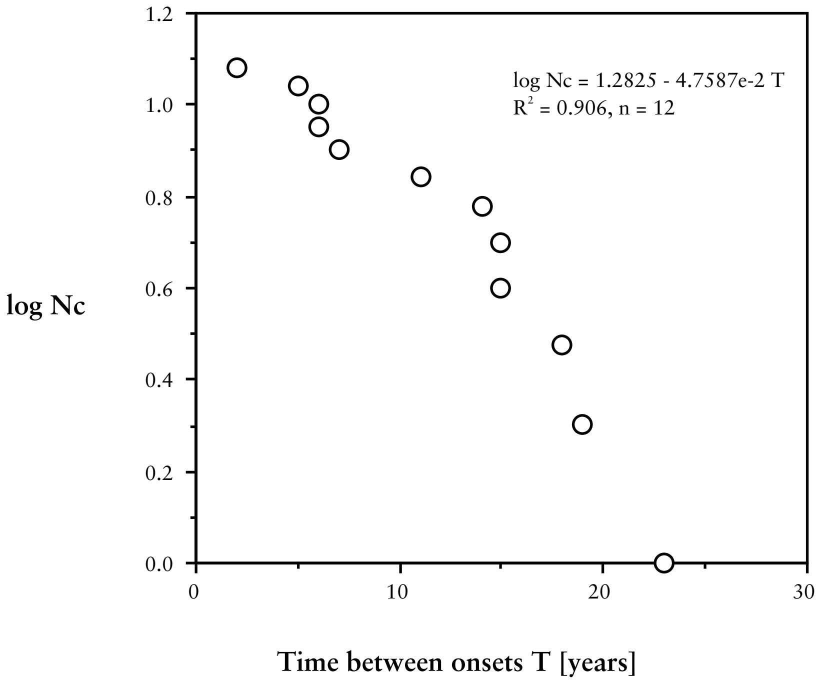

Fig. 3a. Time between onset of largest international wars

obtained for power law models of warfare dimensions *T*, μ, and *D*, that is, equations (1) and (2). The table reports results for each of the five different samples of war cases described earlier in the Methods section. Figure 6 shows the results from the Monte Carlo experiment with uniformly distributed random data.

Section I in table 2 (and fig. 3a–c) reports results for the thirteen most lethal international wars, or top decile of international wars. Section II (and fig. 4a–c) shows findings for the eleven most lethal civil wars, or top decile of civil wars. Section III (and fig. 5a–c) reports results for all twenty-four most lethal wars combined, simultaneously the largest sample examined in this study and the wars in the top decile listed in table 1. For reference, section IV in table 2 provides baseline estimates generated by the Monte Carlo simulation. Recall that *U* obeys a power law with zero fractal slope ($b = 0$) and, consequently, no scaling (n.s.). Note that the R^2 value for the random variable *U* is the lowest (0.77) albeit significant, a clear indication that the coefficient of determination should never be used alone to assess the goodness of fit of a power law.

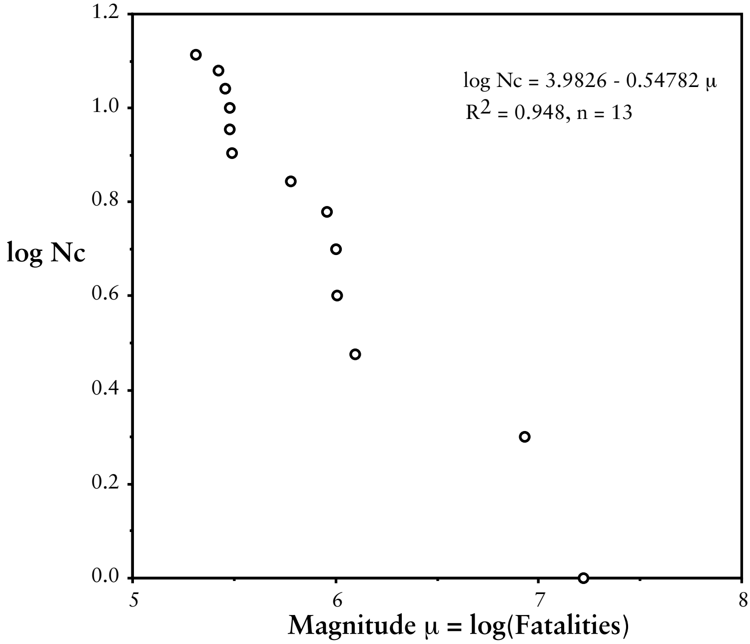

Fig. 3b. Magnitude of largest international wars

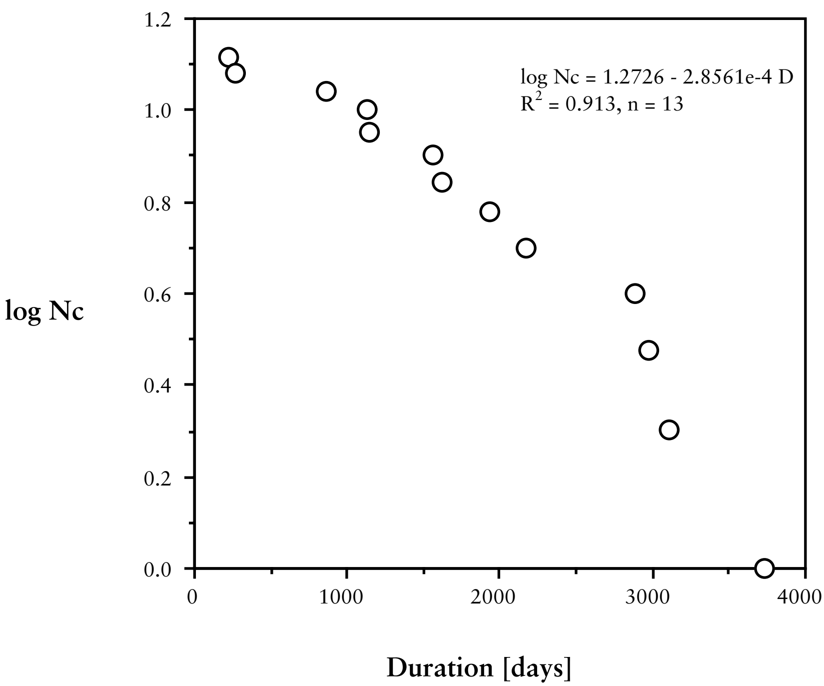

Fig. 3c. Duration of largest international wars

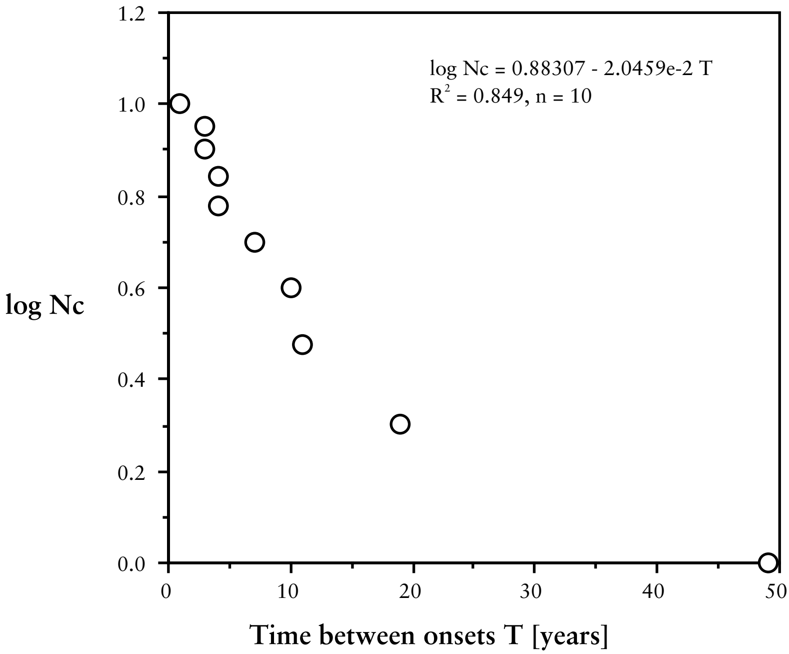

Fig. 4a. Onset of largest civil wars

Section V in table 2 provides an additional set of comparative statistics, consisting of the original scaling parameter estimates first discovered by Richardson (1948, 1960) for cases of "deadly quarrels." Recall that the composition of the Richardson sample (N = 282 deadly quarrels) most closely resembles our third sample, because both combine international and civil wars, although our sample size is smaller (N = 24 highest-magnitude wars).

For each sample (data sets I–V) and warfare dimension (T, μ, D), table 2 also reports the corresponding estimate for the intercept a, the fractal slope b, the sample size N, the variance explained by the power law, or coefficient of determination R^2, the F-ratio, and a reference to the corresponding power law plot (figs. 3–6) for each war dimension. Note that all estimates are OLS and ML, because the linearized form, equation (2), of the power law, equation (1), was used for each dimension. In each sample the estimates for onset time T have $N - 1$ cases, not the original N in the decile, because one case is lost when calculating war outbreaks between consecutive events.

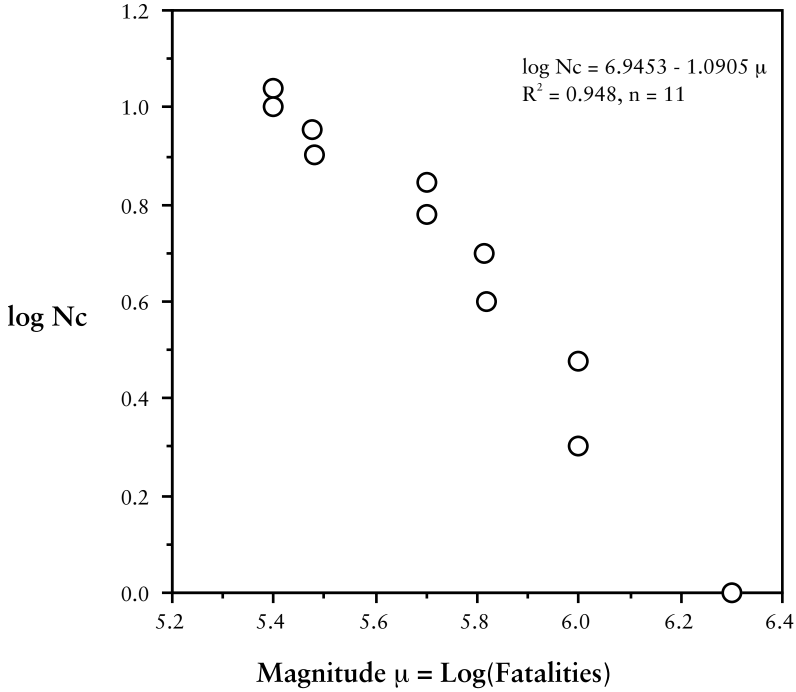

Fig. 4b. Magnitude of largest civil wars

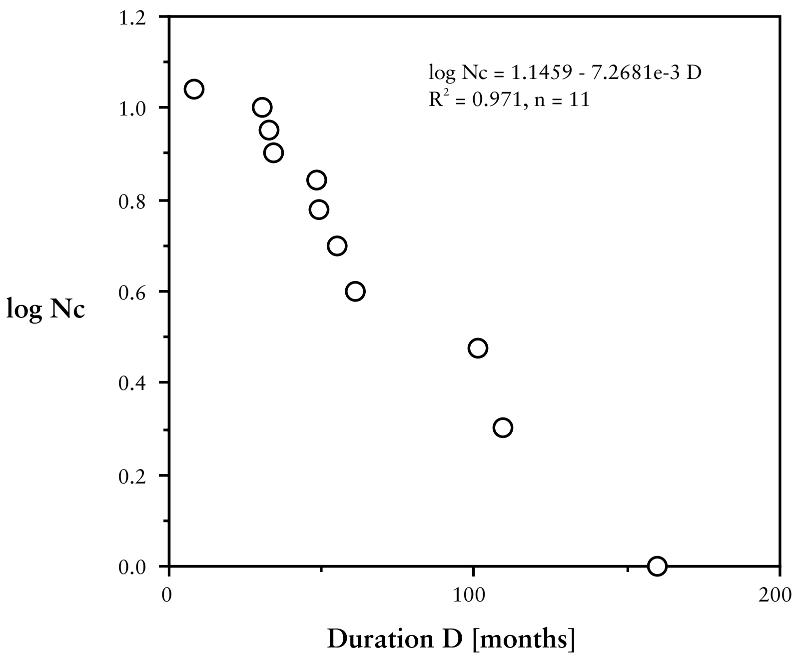

Fig. 4c. Duration of largest civil wars

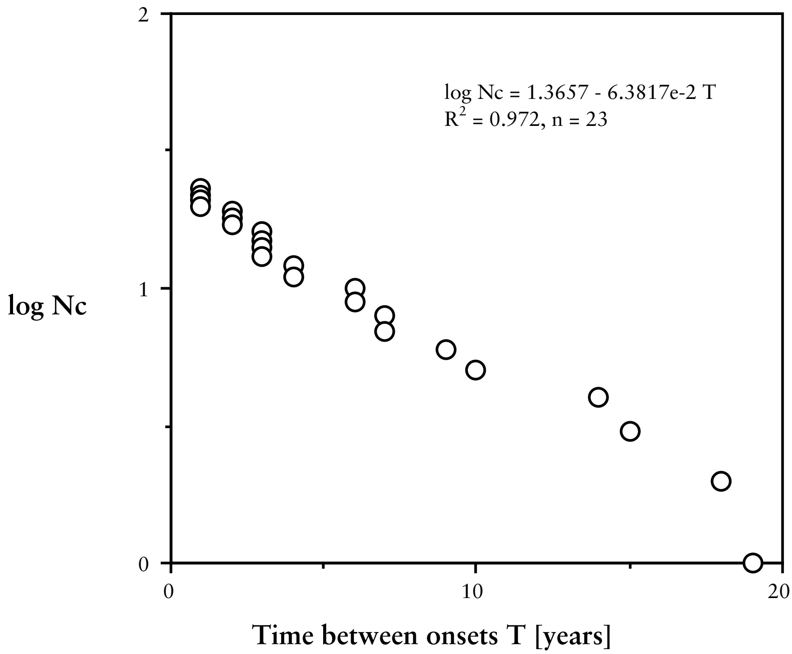

Fig. 5a. Onset of all wars combined (international and civil)

DISCUSSION

In this section we discuss the main results obtained in this study, some implications for current ideas on high-magnitude warfare, and some directions for future research.

Findings

Is high-magnitude warfare governed by power laws? The main findings produced by this study can be summarized as follows.

Power law behavior. Every empirical estimation (table 2, sections I–III) yielded positive results, as shown by the high statistical significance of the estimates. Specifically, all estimates of *a* and *b* (note the consistently high *t*-ratios in parentheses below each estimate), as well as the *F* and R^2 values are significant ($p < .05$), in most cases *highly* significant ($p < .001$). This pattern across different samples of highest-magnitude wars (international, domestic, and combined), as well as different dimensions

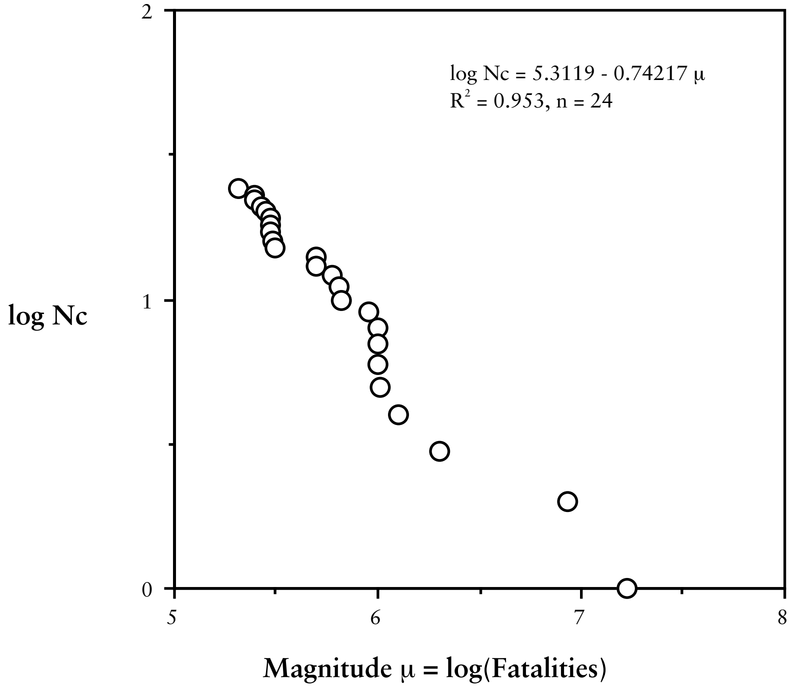

Fig. 5b. Magnitude of all wars combined (international and civil)

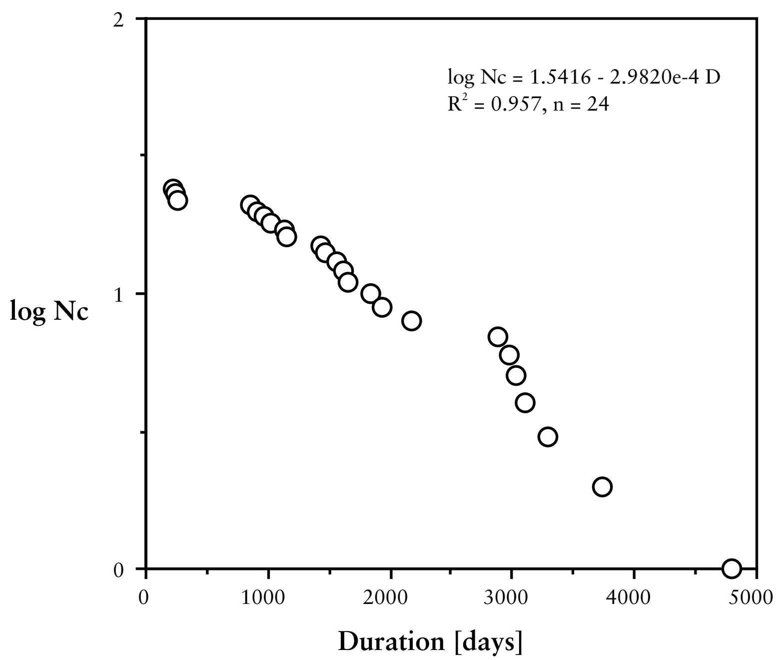

Fig. 5c. Duration of all wars combined (international and civil)

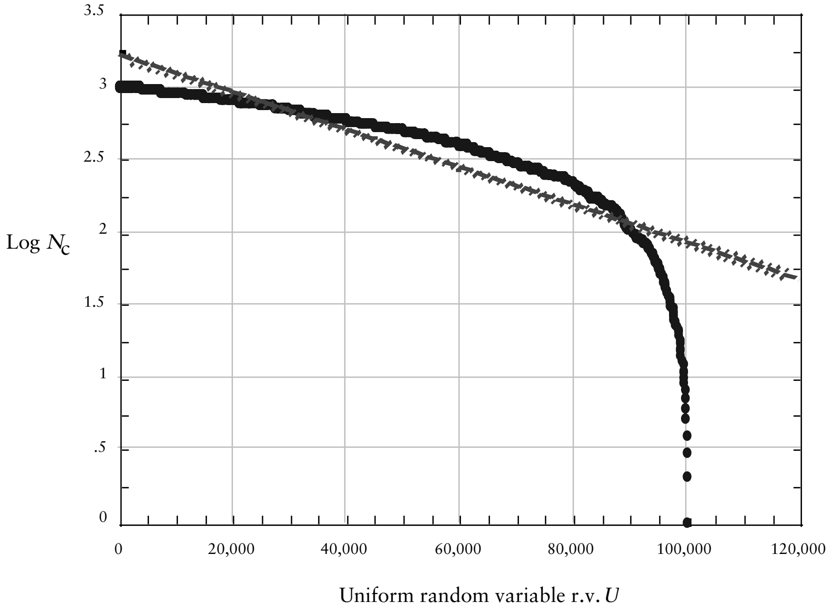

Fig. 6. Power law plot for N = 1,000 synthetic realizations of a uniform random variable $U(0, 100{,}000)$

(onset, magnitude, and duration) provides the strongest confirmation so far for the power law behavior of largest-magnitude warfare.

Yet the parameters for the highest magnitude wars differed from those of the full set reported by Cioffi-Revilla (2000b) (compare figs. 2 and 3–5). This was especially true for fatalities and duration. Why? There are two explanations for this phenomenon. One answer may lie in the "democratization" of war. As a war continues without end in sight, fatalities grow in number, and an increasing proportion of the population is drawn into the war. Germany between 1916 and 1918 is a classic case in point, with its strikes, industrial sabotage, severe food shortages, and naval mutiny. Indeed, it is inconceivable that the Weimar Republic could have emerged in such liberal form (for its time) without the revolutionary sentiments sweeping the country in reaction to the war. National policy during such a high-magnitude war is increasingly affected by large segments of the population that seek to end the war or at least to reduce the casualty level. Wars with potentially shorter durations and lower casualty levels are less subject to such popular influences and are found to the left of the plots in figure 2. Those that have been "democratized" (e.g., also Vietnam) are found on the bottom right of the plot.

Another explanation for the steeper slope of these highest-magnitude power laws lies in the empirical finite size of the international system producing these severe events: there are just so many belligerents, so many possible war alliances, so much armament, so many combat fronts that can be managed simultaneously, and so forth. As a result, the theoretically possible largest magnitudes of warfare are never actually realized due to the underlying finite dynamics.

Warfare magnitude scales. Every set of estimates for the power law of warfare magnitude μ shows a close fit, with highly significant departure from the uniform distribution (compare empirical b slopes for μ with the Monte Carlo slope in table 2, section IV), indicative of strong scaling behavior. This finding is also consistent across samples I, II, and III. Recall that, as noted in the Methods section, the scaling property is not additive—because it is nonlinear—so results from data set III (all wars combined) would not necessarily scale just because results for I (international wars) and II (civil wars) show scaling. This finding therefore replicates and confirms Richardson's original discovery for "deadly quarrels," extending it to all types of high-magnitude warfare measured by the COW data: interstate wars, extrasystemic wars, and civil wars. All high-magnitude wars, not just deadly quarrels, obey the property of self-similarity. Thus, for example, we would expect "major power wars" (Levy 1983), as well as high-magnitude warfare in earlier international systems (Cioffi-Revilla 1991, 1996; Cioffi-Revilla and Lai 1995, 2001; Eckhardt 1992) to follow similar scaling patterns with respect to magnitude.

Onset and duration have weaker scaling symmetry. For both onset and duration the fractal slope estimate is closer to 0, even if the coefficients of determination R^2 and F are high, meaning that the distributions of onset and duration values are closer to a uniform distribution (somewhat weaker scaling). Recall that a uniform random variable U has fractal slope equal to 0. Thus, the temporal variables of high-magnitude warfare, involving the timing of onset and termination, follow a more haphazard pattern with greater uncertainty or higher entropy. This finding is consistent with earlier studies that have emphasized the stochastic nature of war onset and duration (Cioffi-Revilla 1998; Midlarsky 1981). Interestingly, the stochastic approach to the study of war onset was also pioneered by Richardson (1941, 1945a, 1945b, 1960). Our results can now be used to link extant probabilistic models with these new scaling models from complexity theory.

Implications

What does the power law behavior of highest-magnitude warfare imply? The preceding results, together with the concepts from complexity theory discussed earlier, hold the following new implications for high-magnitude warfare in the international system.

Emergence. The power law results from this study hold in the aggregate, regardless of the individual type of war (international or civil), the specific epoch of occurrence (in this case nineteenth century or twentieth century), the identity of participant actors (major powers or minor powers), the nature of decision making involved (rational or not), or other individual characteristics (for instance, weapons technology or firepower). Power law behavior is a global, emergent property of the class of high-magnitude wars. Significantly, this property supports an early claim by J. David Singer, other collaborators in the Correlates of War Project (Singer 1961b; Singer and Small 1972), and other independent researchers (Horvath 1965; Horvath and Foster 1963; Weiss 1963; Wesley 1962) upholding the autonomy of systemic-level theories and models, independent of lower-level explanations. The existence of power laws for high-magnitude wars strongly supports such a claim.

Evolution. The bending of the curves in figure 1 and different power law parameters found in figure 2 suggest an evolutionary pattern to modern warfare. The highest magnitude wars in table 1 (ranked by magnitude) are almost exclusively twentieth-century wars. Indeed, of the top fourteen wars in that table, only two occurred in the nineteenth century. Power law behavior, or what is essentially the same thing—fractal patterns of expansion—have been associated with the rise of states and, under somewhat different conditions, with their dissolution (Midlarsky 1999). Thus, the temporal evolution of warfare to higher magnitudes parallels other societal processes, also intrinsic to state behavior. Perhaps this evolutionary process may have reached a critical point in mutating to such a highly destructive level that it may have become all but obsolete, especially among major powers (Mueller 1989).

Long peace. Some researchers have found the recent "long peace" remarkable (Kegley 1991). However, given the property of scaling for the onset T of high-magnitude warfare, as demonstrated by results in table 2 (sections I, II, and III), it follows that every now and then there *must* be a very high value (realization) of T, or long peace between high-magnitude onsets. The power law predicts this phenomenon in terms of scaling and self-similarity, given that high-magnitude warfare conforms to power laws. For high-magnitude international wars (fig. 3A), our model predicts an upper bound $T_{max} \approx 25–30$ years, meaning that this is the longest peace that can be expected for this kind of warfare.

Hierarchical equilibrium. Our results parallel earlier findings regarding the hierarchical equilibrium nature of warfare in the international system, at least in recent epochs (Midlarsky 1988). Scaling is a form of hierarchical equilibrium. Conversely, hierarchical equilibrium also scales.

Early warning and conflict management. Another concern of the Correlates of War Project has been the design and calibration of early warning (EW) indicators. Our results produce some progress in this area, given the strong scaling patterns reported in this study. More specifically, the onset, magnitude, and duration patterns demonstrated in this study can be used in conjunction with EW indicators derived from probabilistic studies, such as distribution moments and hazard force models (Cioffi-Revilla 1998). For example, based on the ratio a/b from the estimates in table 2, or by finding the intercept $(x_{max}, 0)$ of the fitted lines with the horizontal axes in figures 3–5, it is possible to project maximum values of magnitude and duration for each type of high-magnitude event. Estimates of maxima can then be used in calculations of risk assessment and emergency mitigation preparedness. Although preparedness policies may be futile in the case of purely international events, international agencies may be able to profit from such assessments in the case of high-magnitude civil wars. Our theoretical analysis indicates that civil wars yield a maximum of $\mu_{max} \approx 6.4 \approx 2.3 \times 10^6$ fatalities, just slightly higher than the 1860 Chinese (internationalized) civil war. Beyond such a level we would observe a violation of the power law, which is unlikely. Confidence intervals can also be calculated from table 2.

Further Research

This study suggests a number of potentially fruitful research directions, given the nature of high-magnitude warfare and power laws.

Long-range data. An important extension of power law analysis is to long-range warfare data covering earlier historical periods and a greater variety of belligerents (Cioffi-Revilla 1991, 1996, 2000; Midlarsky 2000a). When did high-magnitude warfare begin to scale? What were the characteristics of the first systems of belligerents that produced such phenomena? What is the relationship, if any, between the scaling pattern of warfare and other long-term social and environmental processes? These and other research directions are being actively investigated in the Long-Range Analysis of War (LORANOW) Project, which will shed new light on the power laws of high-magnitude warfare, especially when compared with parallel results obtained for modern data.

Systematized mass murder. Genocide, a topic not often examined systematically, may be explicable in part by extensions of this type of analysis. Genocides most often occur in tandem with high-magnitude warfare. Is it possible that such genocides also scale (Midlarsky forthcoming)? Future research on long-range patterns of warfare may reveal that distinct possibility.

Theoretical analysis. A variety of theoretical implications can be derived from equations (1) and (2), none of which can be addressed here due to space limitations. For example, different values of the fractal slope b hold different implications for the self-similarity property, as could be demonstrated by calculating the wavelet transformations of each series in the COW or LORANOW data. Another direction for future theoretical research is the relationship between equivalent probabilistic and scaling treatments of the same class of high-magnitude wars. For example, the relationship between the power laws given by equations (1) and (2) and the corresponding set of hazard force equations is not intuitive, but such a link should exist and is important for a better understanding of the underlying dynamics of extreme events such as high-magnitude wars. This type of formal theoretical analysis can be especially fruitful and insightful when founded on empirically tested models, as is now increasingly the case for power laws of warfare.

A driven-threshold-systems conjecture. Cioffi-Revilla and Rundle (1999) have conjectured that wars and other large-scale events in a driven-threshold system (DTS), particularly high-magnitude wars, represent extreme events or coherent structures characteristic of a multiscale system in nonequilibrium conditions. Accordingly, a high-magnitude war, such as an event $\mu > 5.0$ in the COW data, is caused by a critical phase transition, which in turn results from the nucleation of a high-magnitude metastable state when the DTS enters a bifurcation set. Onset of the extreme event is caused by the growth of space-time correlations that can be observed in macroscopic COW data. Such a DTS theory would provide a new dynamic explanation for the occurrence of scaling in warfare.

Computational modeling and simulated data. Recent advances in agent-based simulation models of international processes (Cederman 1997, 2001; Hoffmann 2003; Min, Lebow, and Pollins 2003) will soon permit in-depth comparative analysis of similarities and differences between empirical data and synthetic or simulated data. Do agent-based simulations of international processes wherein warfare, conquests, disintegration, and other phenomena occur also give rise to power laws and scaling? If not, what would be required in terms of additional rules to observe the type of scaling behavior that we have demonstrated for warfare in the real world? If scaling does occur in such simulations, to what extent does it compare with the known features of empirical scaling patterns?

These and other puzzles in the research frontier of the scientific study of war await future investigation. No doubt the Correlates of War Project data, as well as many of its concepts, hypotheses, and methods, will continue to play a key role in advancing our understanding of the causes of war and the conditions for peace.

SUMMARY

This study investigated the scaling and fractal properties of highest-magnitude warfare in the international system, as measured by the Correlates of War data files on international and civil wars. After describing the general characteristics of power laws and defining the relevant class of extreme events—wars in the top decile of the intensity distribution in terms of fatalities—we explained our empirical procedure for testing power laws on COW Project war data. Our findings demonstrated the strong presence of power laws across all types of high-magnitude wars (international wars, civil wars, and wars in general) for three different dimensions of warfare (onset, magnitude, and duration). Different parameters for the total set of wars, on the one hand, and highest-magnitude wars, on the other, reflect both the "democratization" and "finiteness" of war in the latter category and the evolution of warfare to virtually unsupportable levels. These findings therefore replicate Richardson's original discovery of magnitude scaling for "deadly quarrels" and extend that discovery to a more diverse set of conflicts and different dimensions of warfare, not just magnitude. The fact that warfare shows significant temporal-magnitude scaling holds not just intrinsic importance as a general covering law, in the sense of Hempel, but also has a set of implications on emergence, the so-called long peace phenomenon, and conflict management and mitigation policies.