Chapter 11

Testing the Models

A TEST OF A MATHEMATICAL MODEL is made when the predictions of the model are compared with observed results. When data are quantified, results are obtained as numbers. The model, however, as a rule, predicts the results in the form of mathematical expressions involving variables and parameters. These are designated by letters, not numbers. The expressions are evaluated as numbers when specific values are assigned to the letters. These letters stand for two kinds of quantities, *variables* and *parameters*.

In the classical formula for the distance passed by a falling body, $s = at^2$, t, i.e., time, the independent variable, is given a value, which determines the value of s, i.e., the distance fallen, provided a is also given. The latter quantity is a parameter. Its value is, as a rule, not known when the formula is derived. Therefore, a test of the formula can be put in the form of a question of the following sort: "Of all the parabolas of the form $s = at^2$ (every value of a determines a particular parabola) is there one which fits the observed plot of s against t? If so, what is the value of a which determines it?"

Note that two questions are involved. The first one asks whether the data *can be made* consistent with the theory. We say "can be made consistent," not "are consistent," because in our parameter we still have a degree of freedom left. We *seek* a parabola to fit our data. The second question provides further theoretical leverage. If under different conditions the values of a (which give a fitting parabola) are different, we can go on to the next order of questions, namely on what does a itself depend. For example, if the experiments with falling bodies are performed on inclined planes, a will depend on the inclination of the plane. Specifically $a = \frac{1}{2}g \sin \alpha$, where α is the angle which the plane makes with the plumbline and g is another parameter, which, in turn depends on the geographical location and the altitude of the experimental site. Thus, the investigation need not end with a corroboration of the proposed mathematical model. Corroboration is obtained when the value(s) of the parameter(s) is (are) found which effect agreement between the theoretical formula and the data. But this opens the road to the further development of the theory, namely an *interpretation* of the parameter(s) and questions concerning their dependence on the conditions of the experiment.

All this makes good sense if the parameters are few and easily interpretable. In the case of bodies falling in vacuum, there is only one parameter and it is easily interpretable as twice the acceleration of the body. If, however, there are several parameters, the situation is not so straight forward. The parameters may not be easily interpreted and even if they are, the further development of the theory may become so involved that hardly any clear understanding emerges from it. Most important, the presence of many parameters leaves many degrees of freedom in fitting the theoretical formula to the data. Given enough parameters, the theoretical curve can be "twisted" to fit almost any set of data. But then, many other theoretical curves involving the same number of parameters can be made to fit the same set of data, and so the theory is left ambivalent.

In some of our models of Prisoner's Dilemma we find ourselves in situations of this sort. For example, the "full dress" stochastic learning model of the second order, that is, applied to the state-conditioned propensities, x, y, z, and w involves, as we have seen, 256 independent parameters. Aside from the almost impossible task of estimating this number of parameters from a given set of data, it is probably the case that a set of values can *always* be found to fit the data on hand, because of the tremendous number of degrees of freedom. But there may be a very large number of such sets of values which will give equally good fits. And even if we were able to select the "best" set of values, we would be powerless to continue the development of the theory, i.e., to investigate the dependence of every one of the 256 parameters on experimental conditions, which is the standard next step in a theoretical investigation instigated by a mathematical model.

Therefore, when we speak here of "testing our models," we do not mean it in the strict sense of corroboration or refutation, except in those instances when we shall be in a position to refute a model definitively. We shall go only part of the way in putting some models to tests. Our goal will be to gain some understanding of what *may* be happening in repeated plays of Prisoner's Dilemma. In short, the main purpose of the discussion in Part III will be to generate hypotheses rather than to corroborate or refute them.

The "Two Simpletons" Model (Static)

Consider the model of the two tempted simpletons, who are characterized by $y_i = z_i = 0$; $w_i = 1$ and who adjust their x_i so as to maximize expected gain. The formula for the expected gains given in Chapter 8 [cf. Equations (73) and (74)] applies to the simplified case where $S = -T$, $P = -R$. In our games the first of these relations always holds but not the second. To apply the model to our data we therefore need a generalized formula involving three parameters, T, R, and P. The generalized formulae for expected payoffs are given by

$$G_1 = \frac{R + T(x_2 - x_1) + P(1 - x_1x_2)}{2 - 3x_1x_2 + x_1 + x_2}, \qquad (109)$$

$$G_2 = \frac{R + T(x_1 - x_2) + P(1 - x_1x_2)}{2 - 3x_1x_2 + x_1 + x_2}. \qquad (110)$$

Setting $\partial G_1/\partial x_1 = \partial G_2/\partial x_2 = 0$ and solving for x_1 and x_2, we have

$$x_1 = x_2 = x^* = \frac{2T - 3R - P \pm \sqrt{28T^2 + 2RP - 3P^2 + 9R^2}}{6T - 2P}. \qquad (111)$$

Only the larger root has meaning. If $P = -R$, formula (111) reduces to (77) of Chapter 8, as, of course, it should.

Let us now evaluate the right-hand side of (111) for each of our seven games. The result is shown in Table 21.

TABLE 21

Game	x^*
I	.857
II	.980
III	1.14
IV	.912
V	1.20
XI	.989
XII	1.06

According to the theory developed in Chapter 10, x_i should be driven unconditionally toward zero in Games III, V, and XII. In the remaining games there is a threshold value of x, so that x will be driven toward one only if that threshold is exceeded. Unless we know the dynamics of the process, we do not know the actual paths of x_1 and x_2 in the phase space and so cannot say anything about the magnitudes of the average values of these variables. It is reasonable to suppose, however, that the larger the magnitude of x^*, the more quickly and certainly will x be driven toward zero. This conjecture applies even in the cases represented by Games III, V, and XII where x^* exceeds one and hence has no real meaning as an equilibrium. On the basis of this conjecture we would assign the following rank order to the games as the inverse of the rank order of their associated x^* values:

$$I > IV > II > XI > XII > III > V. \qquad (112)$$

But this rank order coincides with that prescribed by our Hypothesis 3 (cf. p. 43).

Therefore the model of the two tempted simpletons who adjust their x's to maximize expected gains can be taken as a model underlying Hypothesis 3.

To be sure, the state-conditioned propensities of the simpletons do not at all correspond to those observed in our subjects. First, the x's of the subjects are practically all large, clustering in the 80's and 90's and this is observed even in the most severe games in which, according to our model, the x's ought to tend toward zero. Further, the y's and z's of our subjects, while not large, are seldom near zero. Finally, the w's of our subjects are consistently low, not equal to 1, as they are assumed in the model. This shows that the model is extremely insensitive to the actual values of the propensities. It does, however, predict the rank order of the games roughly consistent with the observed rank order.

The one discrepancy between the rank order given by (112) and that observed in the Pure Matrix Condition is that involving the rank of Game II, which is observed to be first in the Pure Matrix Condition but ought to be third according to the model. It is natural to conjecture that the reason for this discrepancy is the unrealistically high value of w ($= 1$) assumed in the model.[33]

We wish, therefore, to see what happens at the other extreme, when w is very small. We cannot simply set

$w = 0$, because this would make DD an absorbing state. We can, however, examine the behavior of our system as w *approaches* zero.

As before, we set $y = z = 0$ but now leave w free, assuming $w_1 = w_2 = w$. The steady state Equations (65–68) of Chapter 7 now become

$$CC = \frac{w^2}{(1 - x_1x_2)(1 + 2w - 2w^2) + w^2(1 + x_1 + x_2 - 2x_1x_2)}; \quad (113)$$

$$CD = \frac{w^2x_1(1 - x_2) + (1 - x_1x_2)w(1 - w)}{(1 - x_1x_2)(1 + 2w - 2w^2) + w^2(1 + x_1 + x_2 - 2x_1x_2)}; \quad (114)$$

$$DC = \frac{w^2x_2(1 - x_1) + (1 - x_1x_2)w(1 - w)}{(1 - x_1x_2)(1 + 2w - 2w^2) + w^2(1 + x_1 + x_2 - 2x_1x_2)}; \quad (115)$$

$$DD = \frac{1 - x_1x_2}{(1 - x_1x_2)(1 + 2w - 2w^2) + w^2(1 + x_1 + x_2 - 2x_1x_2)}. \quad (116)$$

Proceeding exactly as before, we calculate G_1 and G_2 and set $\partial G_1/\partial x_1$ and $\partial G_2/\partial x_2$ equal to zero. The parabola corresponding to that given by Equation (75) now becomes

$$w^2[(T + 2w^2T - P)x^2 + (R + 2Rw + P - 2w^2T)x + (w^2T - 2wT - T - P - w^2R)] = 0. \quad (117)$$

Now we see mathematically, as well as intuitively, why we could not set $w = 0$ at the start. If we did, our "parabola" would disappear. As long as $w \neq 0$, we can divide the left-hand side of (117) by w^2 and so obtain the parabola represented by the expression in the brackets. As a check, observe that if $w = 1$, (117) reduces to the numerator of (75).

After dividing by w, we can set $w = 0$, and (117) reduces to

$$(T - P)x^2 + (R + P)x - (T + P) = 0, \quad (118)$$

whose positive root is

$$x^* = \frac{-(R + P) + \sqrt{(R + P)^2 + 4(T^2 - P^2)}}{2(T - P)}. \qquad (119)$$

Inserting the values of T, R, and P from our game matrices, we obtain the results shown in Table 22.

TABLE 22

Game	x^*
I	.61
II	.52
III	.90
IV	.58
V	.98
XI	.74
XII	.73

Now the rank order of the games is the following:

$$II > IV > I > XII > XI > III > V. \qquad (120)$$

Observe that this rank order is exactly that implied by Hypothesis 4 (cf. Chapter 1, p. 43). Thus, we have provided a model for both of our ad hoc hypotheses, from which we had derived the theoretical rank order of the seven games with respect to C. The model is an extremely simple one, involving a trial-and-error adjustment of *one* of the four state-conditioned propensities by each of the players, the other propensities being assumed fixed. The one or the other hypothesis is a consequence of this model depending on the fixed but arbitrary value of w. If w is large, Hypothesis 3 emerges from the model; if w is small, Hypothesis 4 emerges.

It is interesting to observe that the values of x^* are smaller when w is small than when w is large. In other words, x is more likely to be driven to zero when w is large than when w is small. This seems paradoxical at first, since C is positively related to x, and so we have the apparently anomalous result that two "distrusting" players (with low w) will achieve more cooperation than two "trusting" players (with high w). However, this conclusion is unwarranted. Recall that when $w = 1$, there will be fifty percent cooperation even when $x = 0$. When w is near zero, on the other hand, low values of x will induce very large D's. In the last analysis, therefore, players with low w's will also exhibit low C's, as we would expect.

To the extent that the data are in rough agreement with either hypothesis, they are consistent with the model. However, the model cannot be taken seriously as a good representation of the actual situation. First, the hypothesis which is in better agreement with the data, namely Hypothesis 3, is a consequence of the assumption that w is large, whereas in fact w is small. Second, the model implies that in Game V, at least, x is almost certain to be driven to zero whether w is large or small, and this has never been observed. Finally, as has been already pointed out, the fact that the model is of an equilibrium type gives rise to the extremely unrealistic assumption that the players can calculate their expected gains and adjust their conditional probabilities of response accordingly. It does not help to argue that they learn to do so "intuitively," because to get "intuitive" estimates of expected gains and moreover to compare them with average payoffs obtained from previous values of x would require prodigious "intuitive" memories. Therefore, this type of model has at most a heuristic value. It serves as a stage in the trial-and-error process of constructing a theory.

Evidence Against the Four-State Markov Chain

We could view the mass of data yielded by the twenty-one thousand responses in the Pure Matrix Condition as follows: we have here three hundred consecutive responses by each of seventy pairs playing certain (unspecified) versions of Prisoner's Dilemma. Can the massed data be viewed as a realization of a stochastic process to be accounted for by a four-state Markov model? If it can, we shall say that a "composite subject" is playing a "composite game" and is behaving *as if* he were governed by a four-state Markov chain with such and such parameters. If not, we shall look for sources of errors in our assumptions and attempt to construct another model.

The advantage of this deliberately imposed ignorance (pretending that we know nothing about the different subconditions and subpopulations which comprise our total process) is that we can bring in the refinements gradually and stop when we have a model which will have withstood certain specified tests. No model of a real process can withstand *all* tests, and we do not know at this point how far we should go to test our model. It may turn out that a model which is adequate on a certain level will prove to be already sufficiently fruitful in generating interesting hypotheses to serve as the raw materials of a theory on the next level of complexity. This is our principal aim. All models, however refined, will retain an "as if" character. Therefore the level of refinements should correspond to the level of complexity at which the theory becomes fruitful and still remains tractable.

We start with a crude test of the simple Markov model. Suppose we had a population of players each characterized by the same values of the state-conditioned propensities. Our "average player" in the Pure Matrix Condition has the following profile (cf. Table 9): $x = .84$; $y = .40$; $z = .38$; $w = .20$. How would the time courses of the four states look in a population of this sort if they all started out "neutral," i.e., $CC = CD = DC = DD = \frac{1}{4}$? In particular, how long would it take them to reach the steady state?

To see this, we examine a matrix of transition probabilities, constructed from the state-conditioned propensities, as was described in Chapter 7. The matrix is shown below.

	CC	*CD*	*DC*	*DD*
CC	.71	.13	.13	.03
CD	.15	.25	.23	.37
DC	.15	.23	.25	.37
DD	.04	.16	.16	.64

Matrix 16.

Next we square the matrix, square the result, etc., thus getting successively the fourth, eighth, sixteenth powers, etc. When the columns of the matrix are all identical, steady state has been reached, for then, as can be easily verified, Equations (61)–(64) reduce to $(CC)' = (CC)$; $(CD)' = (CD)$, etc.

According to this model, we expect the steady state to be reached at about the thirtieth play. Actually, however, as we have seen, the steady state is reached by the combined time course in the Pure Matrix Condition at only about the 150th trial. Moreover, the discrepancy does not substantially depend on the initial distribution of the states. Whatever this distribution is, the steady state will be reached when the columns of the transition probability matrix become identical. This happens when the transition probabilities are of the sort we observe, much sooner than is observed in the data.

There are at least two possible sources of error in our model Markov chain. One is in the fact that we have lumped seventy pairs into a single process. Each of the pairs, in fact each of the individuals, may be characterized by a different set of propensities (even assuming these to be constant for each individual). It is mathematically not the case that when the propensities of the individuals are lumped into an "average" propensity, the resulting process will be a population average of the corresponding individual processes.

Another possible source of error is the assumption that the propensities remain constant in time. There is a straightforward test of this assumption. Consider the length of a run in some particular state, say *CC*. If the probability that following a given state the next state will be *CC* depends only on the given state (which is the assumption to be tested), then the probability that a given run of *CC*'s will be ended at any specific play will be independent of the number of *CC*'s in the run. To put it in another way, the probability that a *CC* run will have "survived" for at least t plays will be given by

$$p(t) = e^{-mt}, \tag{121}$$

where m is the inverse of the average length of a *CC* run.[34]

Consider the exponent of e. It is a linear function of t. Consider now a generalized form of the negative exponential function, namely $e^{-f(t)}$, where $f(t)$ is a function which is either concave upward, i.e., has a positive second derivative with respect to t, or concave downward, i.e., has a negative second derivative. If $f(t)$ is linear, its second derivative is zero. We can now characterize three types of runs corresponding to the three classes of functions $f(t)$ just described. These are (1) runs which are more likely to be terminated the longer they last (if $f''(t) > 0$); (2) runs which are less likely to be terminated the longer they last (if $f''(t) < 0$); (3) runs which have the same probability of terminating regardless of how long they last (if $f''(t) = 0$). The simple Markov model implies that all runs occurring in the process are of the third type.

We can now see how our runs behave in this respect. Plots of $-\ln p(t)$ against t for Games II and V in the

Pure Matrix Condition are shown in Figures 29 and 30.[35] The curves are unquestionably concave downward. Therefore $f(t)$ is of the second type: the longer a *CC* run lasts the less likely it is to terminate. The corresponding

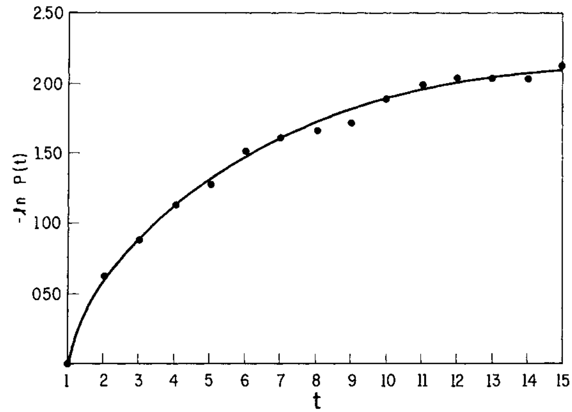

Figure 29. Horizontal: length of *CC* run (t). Vertical: $-\ln p(t)$, where $p(t)$ is the fraction of *CC* runs not yet terminated at t, observed in Game II of the Pure Matrix Condition.

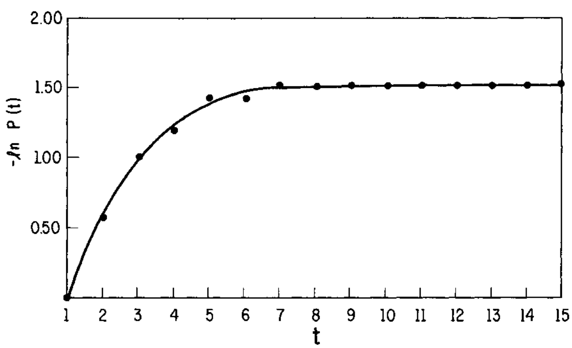

Figure 30. Horizontal: length of *CC* run (t). Vertical: $-\ln p(t)$, where $p(t)$ is the fraction of *CC* runs not yet terminated at t, observed in Game V of the Pure Matrix Condition.

plots of *DD* runs are shown in Figures 31 and 32. The results are similar.

The conclusion that the longer a *CC* run lasts the less likely it is to terminate is an attractive one. It is in accord with our intuitive feeling that continued co-

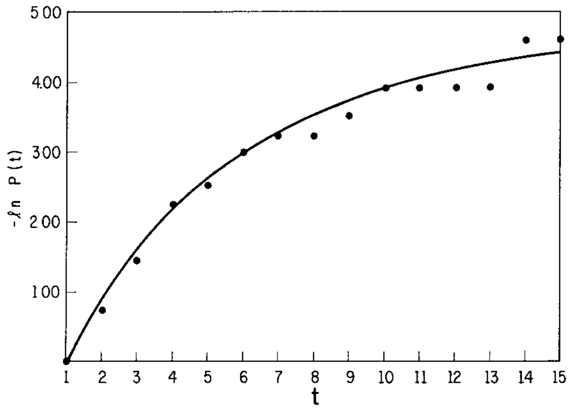

Figure 31. Horizontal: length of *DD* run (t). Vertical: $-\ln p(t)$, where $p(t)$ is the fraction of *DD* runs not yet terminated at t, observed in Game II of the Pure Matrix Condition.

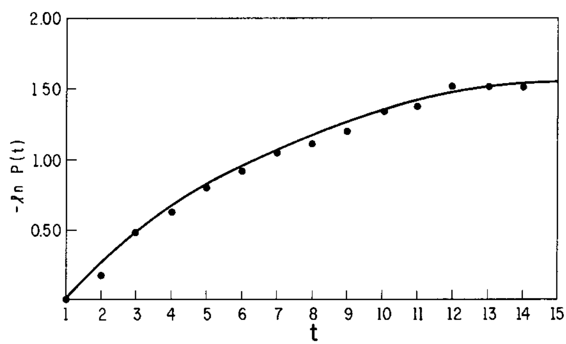

Figure 32. Horizontal: length of *DD* run (t). Vertical: $-\ln p(t)$, where $p(t)$ is the fraction of *DD* runs not yet terminated at t, observed in Game V of the Pure Matrix Condition.

operation reinforces the mutual trust of the subjects and so inhibits defection. Likewise, the same conclusion with respect to *DD* runs can be rationalized on the same grounds. Both conclusions are also in harmony with the previously observed lock-in effect (cf. Chapter 3).

Unfortunately, there is another interpretation of this result, which has nothing to do with self-reinforcement of the double cooperative and the double defecting states. This interpretation is an effect akin to the process of natural selection.

Suppose we have a "population" of processes. Each individual process in the population can end with equal probability at any moment of time. But this probability is different in the different individual processes. If we plot the fraction of the processes which have not yet terminated against time, we shall have some function $g(t)$, which we may write $e^{-f(t)}$, where $f(t) = -\log_e g(t)$. We shall now show that $f(t)$ has a negative second derivative. That is to say, $f(t)$ plotted against time is convex upward, and so the combined process gives the appearance of being "self-enhancing"—the longer it lasts, the less likely it is to end. We shall first show this by a formal mathematical argument and then will offer an informal intuitive one.

Consider a population of processes of type 3, i.e., where $f''(t) = 0$.

The probability that a given individual process has not yet terminated at time t is given by

$$p(t) = e^{-\mu t}, \qquad (122)$$

where μ is the reciprocal of the expected life span of the process. The parameter μ, therefore, characterizes the "mortality" of the process. The bigger the value of μ the larger the "mortality" or the smaller the "viability."

Now the parameter μ is different for the different processes of our population. Let us assume that this parameter is distributed in accordance with some frequency function $\psi(\mu)$. Then the fraction of the processes in the entire population, which have not yet terminated at time t will be given by

$$\int_0^\infty \psi(\mu)e^{-\mu t}\, d\mu, \qquad (123)$$

the function which we have called $g(t)$ or $e^{-f(t)}$. Therefore

$$f(t) = -\log_e \left[\int_0^\infty \psi(\mu)e^{-\mu t}\, d\mu\right]. \qquad (124)$$

Let us denote by I the integral (123). Then

$$f'(t) = -\frac{I'}{I}, \qquad (125)$$

$$f''(t) = -\frac{(I)(I'') - (I')^2}{I^2}, \qquad (126)$$

where the derivatives are with respect to t. We shall have proved our contention if we show that $(I')^2 < (I)(I'')$.

If the frequency distribution $\psi(\mu)$ is reasonably well behaved, differentiation with respect to t can be performed underneath the integral sign. Thus

$$I' = -\int_0^\infty \mu e^{-\mu t}\psi(\mu)\, d\mu, \qquad (127)$$

$$(I')^2 = \left[\int_0^\infty \mu e^{-\mu t}\psi(\mu)\, d\mu\right]^2, \qquad (128)$$

$$I'' = \int_0^\infty \mu^2 e^{-\mu t}\psi(\mu)\, d\mu. \qquad (129)$$

Our inequality can now be written as

$$\left[\int_0^\infty \mu e^{-\mu t}\psi(\mu)\, d\mu\right]^2 < \left[\int_0^\infty e^{-\mu t}\psi(\mu)\, d\mu\right]\left[\int_0^\infty \mu^2 e^{-\mu t}\psi(\mu)\, d\mu\right]. \qquad (130)$$

Now on the right side of (130) we have in the two brackets the averages of $e^{-\mu t}$ and of $\mu^2 e^{-\mu t}$ respectively with respect to the distribution function $\psi(\mu)$. Moreover, the functions $e^{-\mu t}$ and $\mu^2 e^{-\mu t}$ have only nonnegative values. We can therefore assert that

$$\left[\int_0^\infty e^{-\mu t}\psi(\mu)\,d\mu\right]\left[\int_0^\infty \mu^2 e^{-\mu t}\psi(\mu)\,d\mu\right] \geq \int_0^\infty \mu^2 e^{-2\mu t}\psi(\mu)\,d\mu, \quad (131)$$

by virtue of the fact that the product of the averages of two functions assuming only nonnegative values is at least as great as the average of the product. We can therefore say that if inequality

$$\left[\int_0^\infty \mu e^{-\mu t}\psi(\mu)\,d\mu\right]^2 < \int_0^\infty \mu^2 e^{-2\mu t}\psi(\mu)\,d\mu \qquad (132)$$

holds, then inequality (130) certainly holds.

Consider now the function $\mu e^{-\mu t}$. The left side of (132) represents the square of its first moment, $M^{(1)}$, with respect to the distribution function $\psi(\mu)$ while the right side represents its second moment $M^{(2)}$. Now the variance of this function is given by

$$\sigma_m^2 = M^{(2)} - [M^{(1)}]^2. \qquad (133)$$

This variance, however, must be positive except in the trivial case when all the individual processes in our population have the same μ. In all other cases we must have

$$M^{(2)} > [M^{(1)}]^2, \qquad (134)$$

which implies inequality (132) and therefore inequality (130).

This result ought to be intuitively evident. The less viable processes terminate early, and so raise the average viability of the processes which have not yet terminated. The combined process, therefore, gives the impression that it becomes more viable the longer it lasts, but actually the effect is due to a "natural selection" acting on the population.

It follows that if some combined process appears *less* viable, the longer it lasts, it cannot be composed of individual processes, each with a viability independent of time. The best known example of such a compound process is a not too young human population. If we follow this population in time, we see that its overall death rate increases rather than decreases. The common-sense conclusion is that the probability that a particular individual will die at a given moment is not constant but increases with age.

With respect to processes which appear to become more viable, we cannot decide without more evidence whether a process of this sort is a combination of many individual processes with constant but different viabilities or whether for each individual the viability increases (or even decreases) with time. Therefore, all we can say at this point is that the process defined by a run of *CC* or *DD* responses behaves in a way consistent with the hypothesis that for each pair of players the run can terminate at any moment with equal probability. We cannot, however, conclude on the basis of this evidence alone that the hypothesis is true, although we can pursue the consequences of the hypothesis if it is assumed to be true.

Suppose then that our empirical function $g(t)$, i.e., the fraction of runs not yet terminated at time t is actually given by

$$g(t) = e^{-m\sqrt{t}}, \qquad (135)$$

which is roughly indicated by the data.[36] If we suppose that the combined process is the result of lumping all the runs, each one of which has a constant viability (this viability being different in different pairs), we can ask the following question: Is there a distribution $\psi(\mu)$ in the population of pairs which insures this result? In other words, we are asking whether the integral equation

$$e^{-m\sqrt{t}} = \int_0^\infty \psi(\mu)e^{-\mu t}\, d\mu \qquad (136)$$

has a solution $\psi(\mu)$, which is a density distribution, that is, satisfies

$$\int_0^\infty \psi(\mu)\, d\mu = 1. \qquad (137)$$

If we demand also that $\psi(\mu)$ be a continuous function of μ, then we know that (136) has a unique solution. We are not sure, however, whether this solution also satisfies (137). We note that the right side of (136) is the Laplace transform of the $\psi(\mu)$[37]. It turns out that $e^{-m\sqrt{t}}$ does have an inverse Laplace transform which is also a density distribution, namely

$$\frac{m}{2\sqrt{\pi\mu^3}} \exp\left\{-\frac{m^2}{4\mu}\right\}. \tag{138}$$

The question before us is whether (138) is a "reasonable" density distribution of μ, the "reciprocal" of the "viability" of the runs in our population of pairs.

The question cannot be verified directly, since μ cannot be directly observed. We note, however, that the shape of $\psi(\mu)$, as given by (138), is a "reasonable" one. It resembles a logarithmic normal distribution, which is frequently observed with respect to parameters which cannot assume negative values (as is the case with μ). We conclude, therefore, that the "natural selection" principle can be reasonably invoked to explain the observed distribution of lengths of runs.

Nevertheless, in view of the attractiveness of the self-reinforcement hypothesis, we shall not reject it until we have put it to a further test. We shall examine the distribution of lengths of runs in the protocols of individual pairs. If we still observe the same effect in the distribution of the *individual pairs*, namely that the longer the run lasts the less likely it is to terminate, we have a partial confirmation of the self-reinforcement hypothesis, because the natural selection principle cannot be assumed to operate on a single pair.

We note in the protocol of each pair the number of, say, *CC* runs which are at least one play long, at least two plays long, etc. Call these numbers $N(1)$, $N(2)$, etc. Obviously $N(1) \geq N(2) \geq N(3)$, etc. Consider the ratio

$$r_{cc}^{(i)} = \frac{N(i)}{N(i-1)} \leq 1. \tag{139}$$

This ratio is an estimate of the probability that a run will not end with the $(i-1)$th play given that it has lasted for $i-1$ plays. Therefore, the behavior of the $r_{cc}^{(i)}$ averaged over the population of pairs $(i = 1, 2, \ldots)$ will give us an idea of whether the "viability" of *CC* runs increases, decreases, or remains the same on the average in single pairs, i.e., without the operation of "natural selection."

To get the most information, we would like to examine long sequences of the $r^{(i)}$. However, some pairs will not have longer runs, and for these pairs the ratios $r^{(i)}$ will not be defined for $i - 1 > m$, where m is the length of the longest run observed in the pair in question.

We shall therefore resort to the following convention. In the case of *CC* runs and *DD* runs, we shall average the $r^{(i)}$ over all pairs which have runs of this type at least five long. This will give us $r^{(1)}$, $r^{(2)}$, $r^{(3)}$, and $r^{(4)}$. This comprises most but not all of the pairs in our population. Now the pairs which do have, say, *CC* runs at least five plays long are a selected subpopulation of our population. Indeed we would expect these to be the more cooperative pairs. We may find, in fact, that the *CC* runs of these selected pairs do become more "viable" as they become longer.

Next, we adjoin to this subpopulation the pairs that have no *CC* runs at least four plays long. In the combined population we can now estimate only $r_{cc}^{(1)}$, $r_{cc}^{(2)}$, and $r_{cc}^{(3)}$. If the pairs with shorter maximal runs are "less cooperative," we might expect that the increasing viability of the *CC* runs, if it is observed in the originally selected population, becomes weaker in the larger population.

We continue by adjoining the pairs which have no *CC* runs at least three plays long. Now we can estimate only $r_{cc}^{(1)}$ and $r_{cc}^{(2)}$ and expect the "lock-in effect" as reflected in the greater viability of longer *CC* runs to become still weaker. It may even be reversed, in which case we shall observe $r_{cc}^{(2)} < r_{cc}^{(1)}$.

We shall repeat this procedure with *DD* runs and with the unilateral runs. In the case of the latter our original selected population will comprise the pairs which have unilateral runs of four or more, which will give us estimates of $r_{CD}^{(i)}$ (or $r_{DC}^{(i)}$) for $i = 1, 2$, and 3 only. There is no point in estimating $r_{CD}^{(i)}$ for $i > 3$, because the pairs which have unilateral runs longer than four plays are too few in number.

The results are shown in Table 23.

TABLE 23

	$r_{CC}^{(1)}$	$r_{CC}^{(2)}$	$r_{CC}^{(3)}$	$r_{CC}^{(4)}$	$r_{DD}^{(1)}$	$r_{DD}^{(2)}$	$r_{DD}^{(3)}$	$r_{DD}^{(4)}$	$r_{CD}^{(1)}$	$r_{CD}^{(2)}$	$r_{CD}^{(3)}$
(5)	.68	.79	.86	.87	.69	.69	.71	.74			
(4)	.66	.78	.80		.68	.67	.68		.34	.39	.32
(3)	.65	.75			.67	.62			.27	.21	

(5): Average over pairs having runs at least five long.
(4): Average over pairs having runs at least four long.
(3): Average over pairs having runs at least three long.

We observe the following. In the selected pairs which have *CC* runs of five or more plays the lock-in effect is quite pronounced, as evidenced by the steadily increasing $r_{CC}^{(i)}$ ($i = 1, 2, 3, 4$). Moreover, this effect is still strong even when the pairs which have no *CC* runs longer than 3 are adjoined.

With regard to the *DD* runs, the picture is different. The lock-in effect is considerably weaker, and is present only in the pairs which have *DD* runs of five or more plays; further, it disappears when pairs are adjoined with runs not longer than four, and is actually reversed when pairs are adjoined with runs not longer than three.

With regard to the unilateral ("martyr") runs the picture is again a different one. Even in the pairs having such runs of at least four plays, the lock-in effect is no longer observed. The probability that a unilateral response continues after two such responses is, to be sure, greater than the probability that a single response is repeated, but the probability that a run of three *CD*'s (or *DC*'s) continues is smaller. When pairs are adjoined with no unilateral runs greater than three, the reverse of the lock-in effect is observed.

In summary, then, we have ample evidence that, at least in those pairs which have long runs, the probability that a run ends at any time is not a constant. Especially in the case of *CC*, a lock-in effect seems to operate: the longer such a run lasts the more likely it is to continue. In our opinion it is this effect which is responsible for the failure of the four-state Markov chain to describe the time courses of the states.