Chapter 12
Simulation

To TEST A MATHEMATICAL MODEL means to compare the predictions of the model with observations. If the model contains no free parameters, such predictions are categorical. Usually, however, a model involves one or more parameters which are to be "adjusted" to fit the theory to observations. This adjustment is a straightforward matter if the mathematical laws derived from the model are simple. For example, the law derived from the model which assumes constant acceleration is $s = at^2$. Here a is a free parameter, and it can be estimated by a single reading of s and of the corresponding t, namely $a = s/t^2$. In case the data do not fit the formula exactly, a "best estimate" of a is still a simple problem. For example, we can plot s against t^2 and obtain a value of a so as to minimize the sum of the squared deviations of the data from the (theoretical) straight line.

As the number of parameters increases, the task of estimating them becomes progressively more unwieldy and the criterion for the "best fit" becomes more vague.

In principle, one could by trial and error gradually approach a set of values for the parameters which will generate data to agree with the real data to a greater and greater extent, i.e., with respect to an ever greater number of statistics. If our object were to find the "best" model of Prisoner's Dilemma, we would do just that and, in addition, go through the parameter-adjusting process of several different models. Here

the job has been only barely started. In particular we shall obtain simulations from (1) the Markov chain model with fourteen states (including two absorbing states) described on pp. 123 ff; (2) the stochastic learning model of the first order described on pp. 138 ff.

We started each run from the same initial condition (i.e., equiprobable *C* and *D* responses) and chose the parameters so as to fit the time course of *C* averaged for the entire Pure Matrix Condition. We then compared the time courses of the four states (*CC*, *CD*, *DC*, and *DD*) obtained from the simulation with the data obtained from that condition, and the variance of *C* obtained from the simulation and that observed in the same data.

The comparison of the time course of *C* obtained from the Markov model with absorbing states and the data is shown in Figure 33. The parameters x, y, z, w, γ, and δ were adjusted by trial and error until a reasonably good fit was obtained. Note that it has been possible to obtain the initial decline of *C* and its recovery by this simulation. Evidently what is happening in the simulated process is the following. Because of the small values of γ and δ, hardly any of the simulated pairs pass into either of the absorbing states in the early stage. On the other hand, the values of x, y, z, and w are such that *C* initially decreases. However, eventually the pairs start to pass into the absorbing states. Since the probability of passing into ΓΓ is larger than that of passing into ΔΔ, recruitment into the *CC* lock-in is more rapid. Since this recruitment is irreversible, a "recovery" is observed in the average *C* as more pairs get into the ΓΓ absorbing state. This recruitment is just sufficiently great to offset the downward trend of *C* due to x, y, z, w, and δ to match the gradual growth of *C* observed in the data. In this way a fair fit of the time course of *C* is obtained.

Let us now see how good the resulting fits are for the time courses of the four states. The comparison is shown in Figures 34–36.

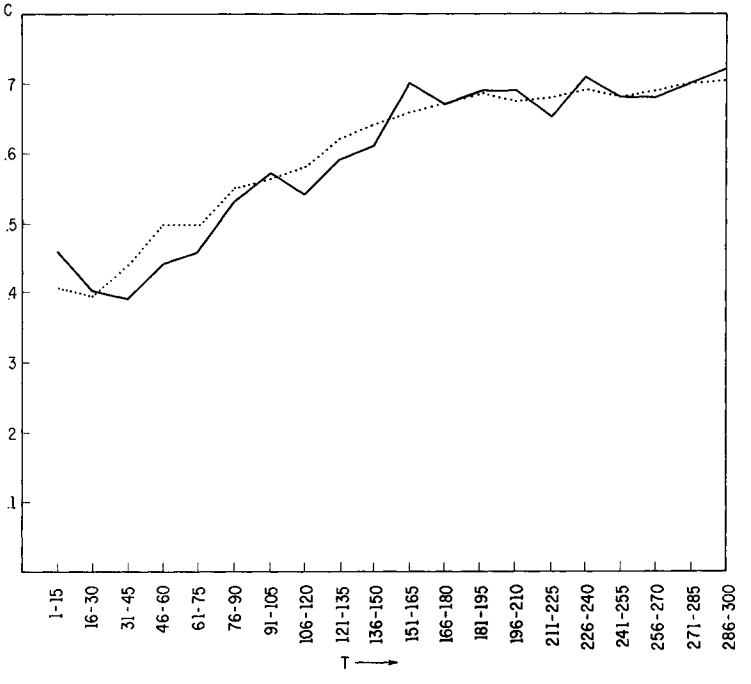

Figure 33. Comparison of observed time course of C in the Pure Matrix Condition (solid line) with that obtained from a simulated Markov model (dotted line) with absorbing states. Parameter values: $x = .6$; $y = .43$; $z = .35$; $w = .30$; $\gamma = .018$; $\delta = .0024$.

As we see, the simulated time courses of the four component states show systematic discrepancies when compared with the observed time courses. Specifically, the frequencies of both the *CC* and the *DD* responses are too low in the simulation while those of the unilateral responses are too high. This suggests that if

we had used higher values of both γ and δ, thus insuring more frequent lock-ins on both *CC* and *DD*, we might have obtained better fits for the three component time

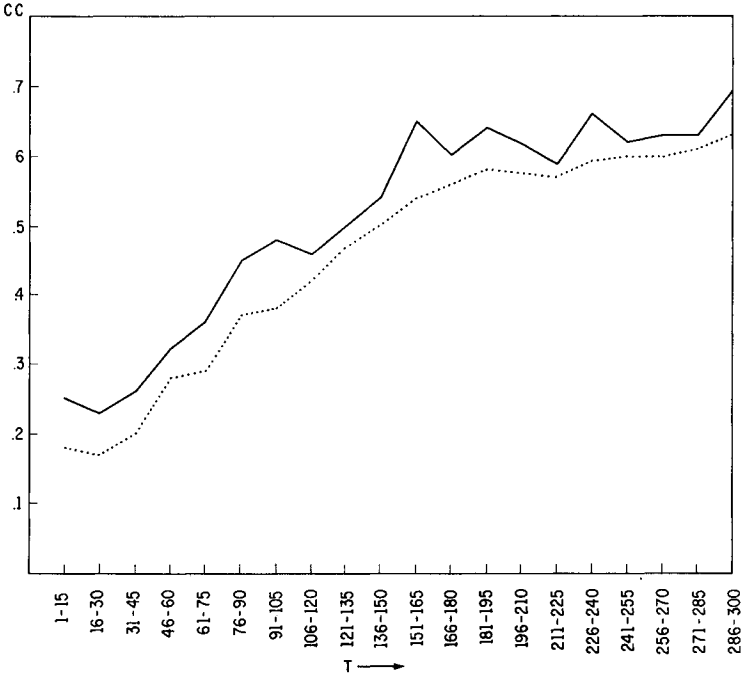

Figure 34. Comparison of observed time course of *CC* in the Pure Matrix Condition (solid line) with that obtained from a simulated Markov model (dotted line) with absorbing states. Parameter values: $x = .6$; $y = .43$; $z = .35$; $w = .30$; $\gamma = .018$; $\delta = .0024$.

courses. However, we followed the policy of stopping the search for the best fitting parameters once the composite (*C*) time course was reasonably well fitted by the simulation and so did not attempt to get better fits for the components by further manipulation of the parameters.

Turning to the first order stochastic learning models, we have the comparisons between simulated and observed time courses shown in Figures 37–40.

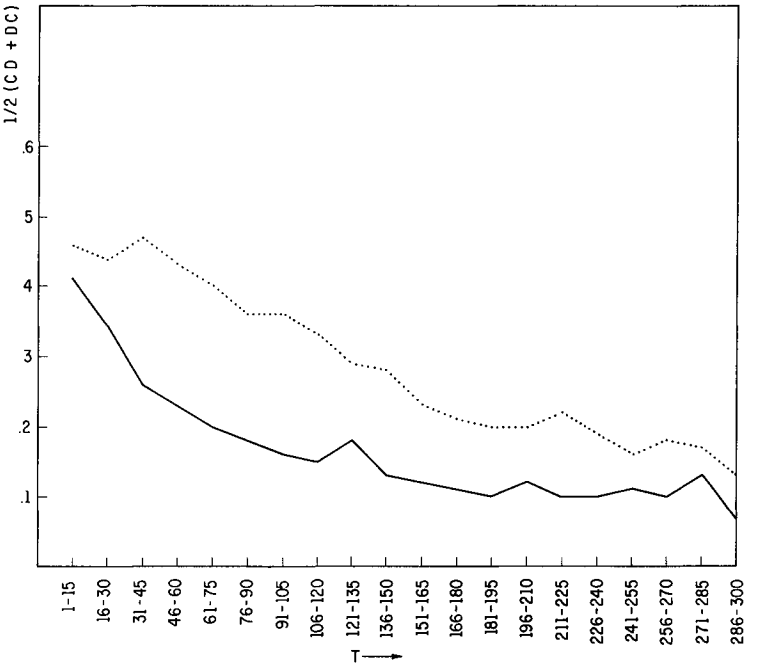

Figure 35. Comparison of observed time course of 1/2 $(CD + DC)$ in the Pure Matrix Condition (solid line) with that obtained from a simulated Markov model (dotted line) with absorbing states. Parameter values: $x = .6$; $y = .43$; $z = .35$; $w = .30$; $\gamma = .018$; $\delta = .0024$.

As is seen from the figures, the fits are reasonably good for all the components although, as before, the parameters were fixed as soon as a good fit was obtained for the composite curve.

Finally we compare the variances of the variables C, CC, CD, DC, and DD as they emerge in the seventy simulated pairs of each model with the variances observed

in the seventy real pairs of the Pure Matrix Condition. The comparison is shown in Table 24.

We see that the variances obtained in the simula-

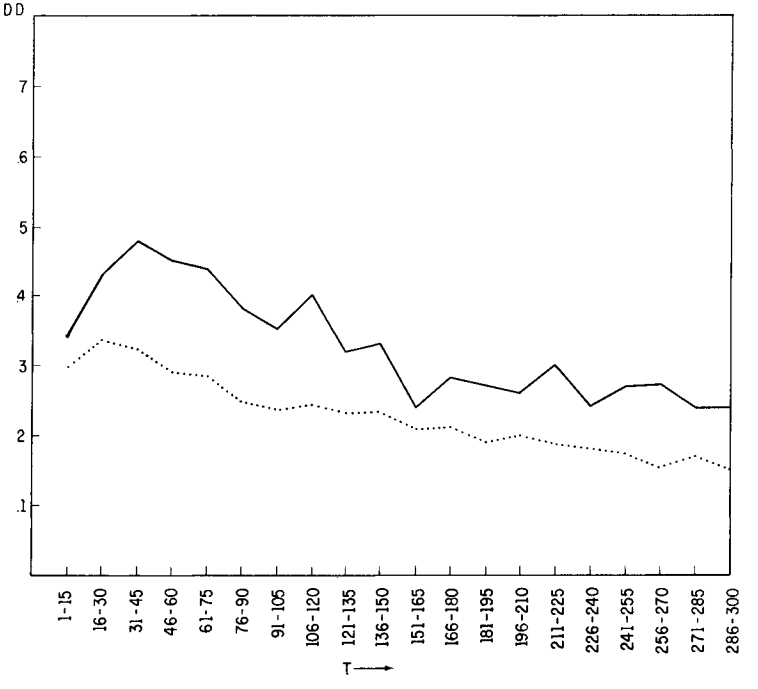

Figure 36. Comparison of observed time course of *DD* in the Pure Matrix Condition (solid line) with that obtained from a simulated Markov model (dotted line) with absorbing states. Parameter values: $x = .6$; $y = .43$; $z = .35$; $w = .30$; $\gamma = .018$; $\delta = .0024$.

TABLE 24

	Var [*C*]	Var [*CC*]	Var [*CD*]	Var [*DC*]	Var [*DD*]
Pure Matrix Condition	.091	.110	.006	.006	.080
Markov Model with Absorbing States	.069	.091	.014	.014	.056
First Order Stochastic Learning Model	.094	.134	.004	.004	.060

tions are comparable to those observed in the data, the stochastic learning model showing perhaps somewhat better agreement.

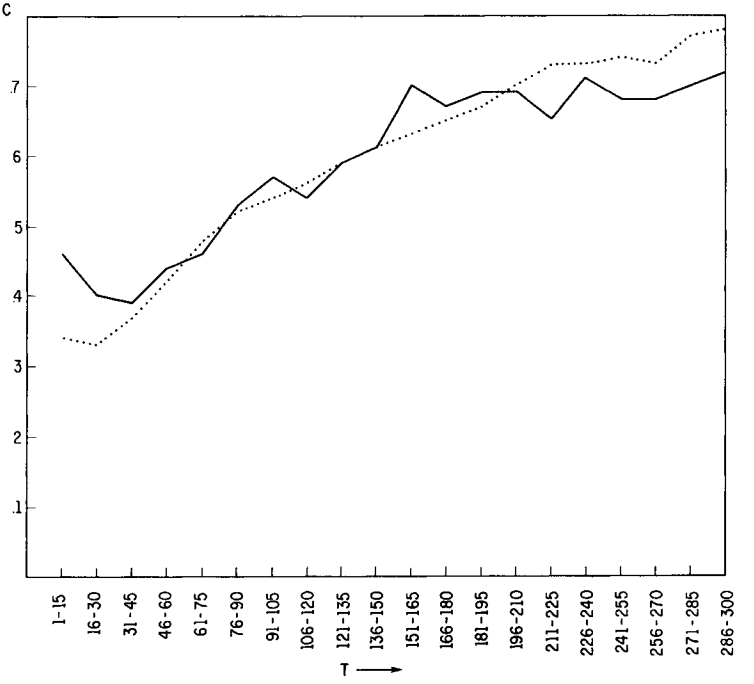

Figure 37. Comparison of observed time course of C in the Pure Matrix Condition (solid line) with that obtained from a simulated first order stochastic learning model (dotted line). Parameter values: $\alpha^{(1)} = .57$; $\lambda^{(1)} = 1$; $\alpha^{(2)} = .4$; $\lambda^{(2)} = 0$; $\alpha^{(3)} = .4$; $\lambda^{(3)} = 0$; $\alpha^{(4)} = .74$; $\lambda^{(4)} = .4$.

Note that the simulated data were obtained from a model in which every "subject" was characterized by the same parameter values. The variances resulted only from the stochastic process itself. This shows that the observed variances need not *necessarily* be ascribed to individual differences among subjects, a point brought out by B. P. Cohen (1963).

We know, of course, that individuals do differ. We know also that the combined data of the Pure Matrix Condition represent the protocols of seven

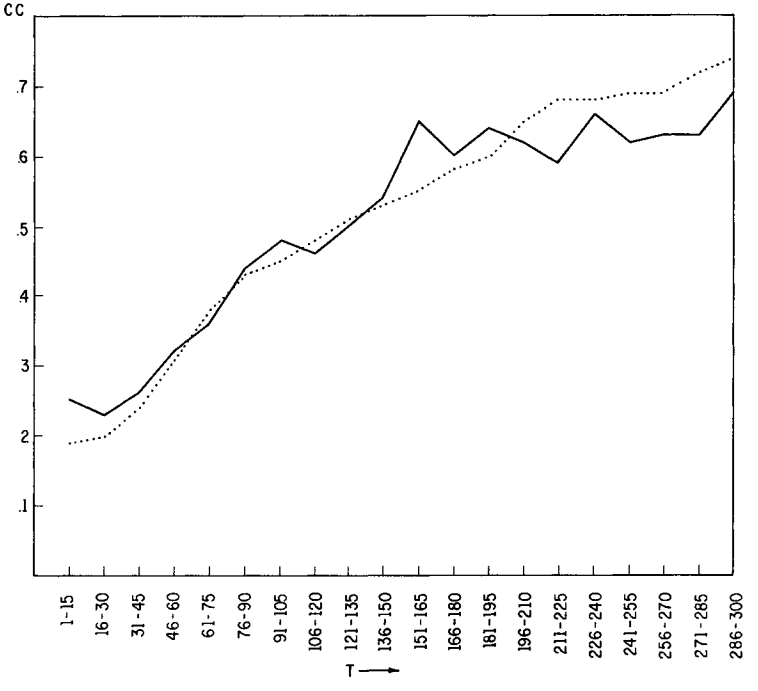

Figure 38. Comparison of observed time course of *CC* in the Pure Matrix Condition (solid line) with that obtained from a simulated first order stochastic learning model (dotted line). Parameter values: $\alpha^{(1)} = .57$; $\lambda^{(1)} = 1$; $\alpha^{(2)} = .4$; $\lambda^{(2)} = 0$; $\alpha^{(3)} = .4$; $\lambda^{(3)} = 0$; $\alpha^{(4)} = .74$; $\lambda^{(4)} = .4$.

different games, among which significant differences in performance have been observed. These credible sources of variance are deliberately ignored in our models. It is important to keep in mind that *every* model is an "as if" representation of reality. Physicists talk, for example, about the "effective volume" of a molecule, meaning not its real volume but the volume of

an object with given properties which behaves like the molecule in question in certain contexts.

Similarly the parameters of our simulated models are "effective parameters," i.e., parameters of a hy-

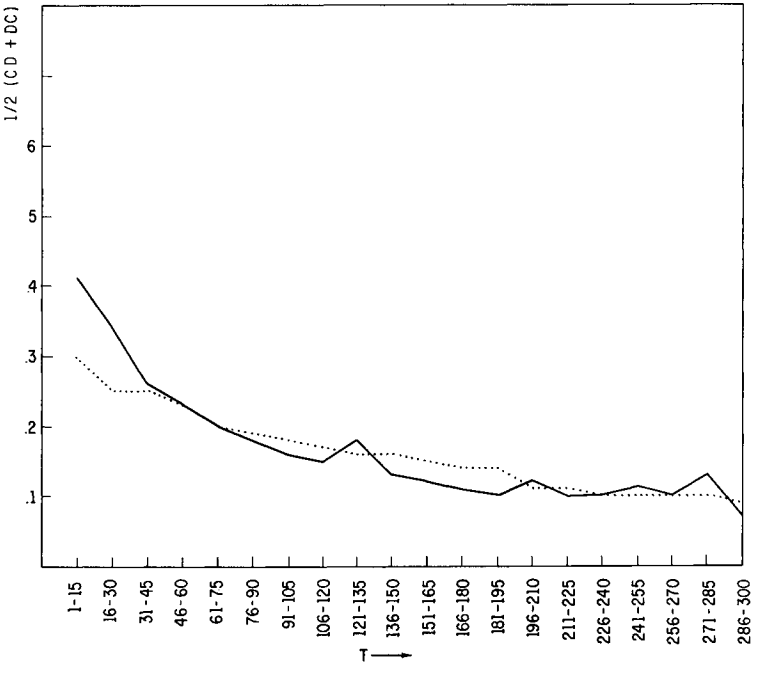

Figure 39. Comparison of observed time course of 1/2 $(CD + DC)$ in the Pure Matrix Condition (solid line) with that obtained from a simulated first order stochastic learning model (dotted line). Parameter values: $\alpha^{(1)} = .57$; $\lambda^{(1)} = 1$; $\alpha^{(2)} = .4$; $\lambda^{(2)} = 0$; $\alpha^{(3)} = .4$; $\lambda^{(3)} = 0$; $\alpha^{(4)} = .74$; $\lambda^{(4)} = .4$.

pothetical subject "replicated" 140 times who plays Prisoner's Dilemma "stochastically" and whose protocols are reasonable facsimiles of observed protocols.

It is very likely that if the first order learning model, which gave good results as far as it went, were pressed

further (for example, if its distributions of lengths of runs or some other statistics were compared with data), it too would eventually fail. This would then call for further refinement. The goal of such an investi-

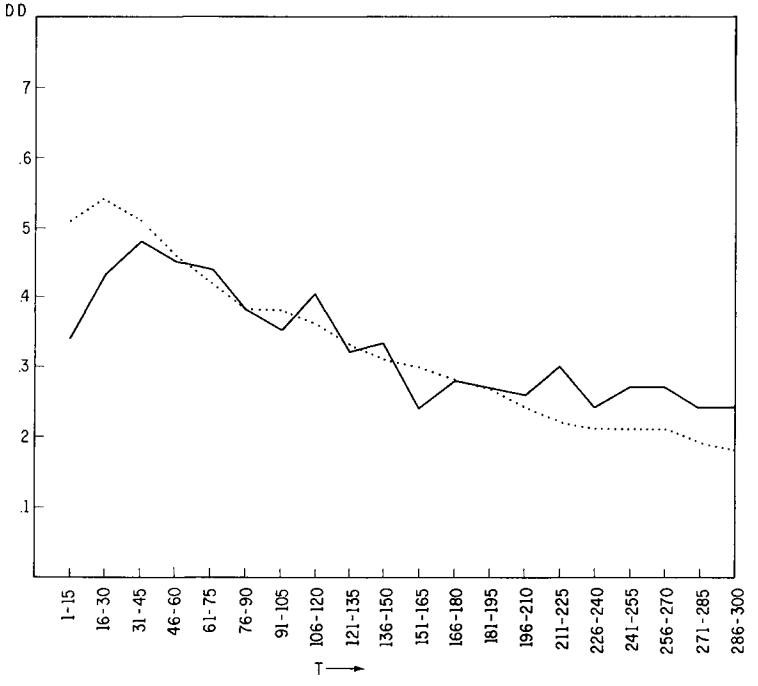

Figure 40. Comparison of observed time course of *DD* in the Pure Matrix Condition (solid line) with that obtained from a simulated first order stochastic learning model (dotted line). Parameter values: $\alpha^{(1)} = .57$; $\lambda^{(1)} = 1$; $\alpha^{(2)} = .4$; $\lambda^{(2)} = 0$; $\alpha^{(3)} = .4$; $\lambda^{(3)} = 0$; $\alpha^{(4)} = .74$; $\lambda^{(4)} = .4$.

gation would be to reach a model good in several important respects and then to declare the parameters in that model to be "effective" parameters which describe a Prisoner's Dilemma experiment under given conditions with a given population as subjects. For example, if we were to declare the first order learning

model adequate, the parameters we would be dealing with would be the four learning parameters $\alpha^{(j)}$ $(j = 1 \ldots 4)$ and the parameter $\lambda^{(4)}$, which represents the asymptotic value of the probability of a *CC* response following a long run of *DD* responses. A given game, condition, or population would then be scored by the values of these parameters which would give the best fit to its performance.

On the other hand, if the Markov model with absorbing states were declared adequate, then populations or conditions would be scored in terms of x, y, z, w, γ, and δ. The psychological interpretation of these parameters is different from that of the α's and the λ's; consequently the psychological theory that would emerge from the Markov model would also be a different one than one that would emerge from a stochastic learning model.

The direction for future theory construction is thus indicated. One should seek a mathematical model which among many tested models describes most accurately with respect to most variables the behavior of populations playing Prisoner's Dilemma. The parameters of the model, psychologically interpreted, would then become the key concepts of the theory, and the content of the theory would be a collection of statements arrived at by induction or deduction concerning the interrelationships among the parameters, their dependence on manipulable conditions, and their role as characteristics of populations.