Voting by Veto

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In his seminal discussion of public goods, Paul Samuelson (1954) both defined the necessary conditions for Pareto efficiency in the presence of a public good, and emphasized the problem of getting individuals to reveal their preferences for the good. Since all members of the community jointly consume a public good, a market mechanism cannot be relied upon for revealing preferences, and some form of nonmarket mechanism, or voting procedure, appears required. To date, no agreement exists in the literature over which voting procedure, if any, can be efficiently used to reveal individual preferences for public goods. The unanimity rule appears to be the logical choice. If all can be made better off through the provision of a public good, none should oppose. But unanimity has generally been rejected as a useful voting rule—first, because it seems to require a potentially lengthy and costly redefinition of the issue to be decided until one benefiting all members of the group is found; second, because it grants to *each* member a right to veto *every* issue proposed, thus encouraging strategic behavior and the endless defeat of all issues; and third, because the probability of the status quo's becoming the committee decision seems too high, as members favored under the status quo exercise their unlimited veto rights to maintain their positions.[1]

Nor is a plurality rule likely to result in Pareto-preferred outcomes. When fewer than all members of a committee can determine the committee's decision, it is likely that some (presumably among those who oppose) are made worse off by the decision. Thus, a decision which might be formulated in such a way so that all committee members would be better off is likely to pass under a plurality rule in a form in which some members are made worse off. A potentially Pareto-preferred decision seems again unobtainable.

The present essay proposes a voting rule that can, like unanimity, result in Pareto-preferred outcomes, wherever they appear possible, but has the potential for avoiding the other disadvantages of the unanimity rule. It is meant to deal with decisions of a public good type, improvements in allocative efficiency potentially beneficial to all. The bulk of the paper is devoted to a somewhat simpler problem, however, and that is how to distribute a bundle of known benefits, B, among the members of the committee. This problem might seem easily soluble under a variety of voting rules, particularly if the committee risked forfeiting the benefits if they did not agree to a division of them. But, it is also the kind of decision that can lead to an endless cycle of defeated proposals under some form of plurality rule, as each issue proposes a

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new division of the benefits among a coalition capable of defeating the previous winning coalition, but also capable of being defeated itself.[2] It is also the kind of decision which critics of unanimity have envisaged as resulting in the endless defeat of each issue proposed, as some voter(s) always vote against a proposed division, in the hopes of securing a more favorable division on some subsequent winning proposal. Thus, the problem of devising a voting rule to divide a given amount of known benefits is of some interest in and of itself. In addition, it can be regarded as a first approximation to the problem of reaching collective agreement on a public good, the division of the benefits among the voters being a representation of the relative distances moved along each voter's utility axis in choosing a point on the generalized Pareto frontier. The problem of revealing preferences on public goods is taken up in section 3.

Throughout the next three sections, we assume coalitions are not formed. Each voter acts independently to maximize his expected utility. What happens when coalitions are formed and the potential for redistribution under the procedure are discussed in section 4. Conclusions are drawn in section 5. We begin, however, by describing the rule and analyzing its properties for committees of two and three persons in section 1, and then extend the results to committees of n persons in section 2.

1. The Voting Rule—Two- and Three-Person Committees

A committee of two is to decide whether to accept a gift of B dollars or not, and if so, how to divide it. The status quo is to reject the gift, $S(0,0)$. The set of Pareto-optimal decisions is to accept the gift and divide it in the proportions $(x, 1 - x)$, $x \geqq 0$.

Now consider the following voting procedure. Each voter can propose one division of B. The issue set is then composed of the two voters' proposals and the status quo, $A = \{P, P', S\}$. An order of voting is picked at random. The voter randomly selected to vote first removes one issue from the set. The second voter removes another. The issue remaining is the committee decision.

Each voter makes two decisions: the proposal of one issue for inclusion in the issue set, and the removal of one issue from this set. The voting procedure can be conveniently analyzed by considering each voter's strategies at the two decision points.

It does not benefit a voter to propose an issue that makes the other voter worse off than under the status quo. If such an issue were proposed, the other voter would certainly remove it from the issue set. Thus, redistribution, other than of the Pareto-efficient kind in which both givers and receivers are better off, cannot be achieved by this rule.[3] Each voter can benefit by his choice of proposal only if he chooses one which the other voter prefers to the status quo. Then, given a choice between the status quo and one voter's proposal, the other can be expected to propose an issue benefiting the other voter and himself, perhaps, even more.

Suppose the voter's proposals are $P(B - 1, 1)$ and P' $(1, B - 1)$, and further, that the proposer of P' is chosen by lot to vote first. He rejects P and the other voter must choose between P' and S. Since he is somewhat better off under P', he rejects S making P' the committee decision. The advantage in position lies in voting first. The second to vote is forced to choose between the status quo and the other voter's proposal.

Before turning to the three-person case, two observations are in order. First, the voting outcome must be Pareto-optimal. Each voter has both an incentive to make a Pareto-optimal proposal, and to reject the status quo in favor of the other voter's Pareto-preferred proposal.[4] Second, the procedure is fair. Each voter has an equal chance of going first.

When there are three members of the committee, an additional procedural decision arises. The full sequence of veto voting in the second step can be revealed to all voters once it has been determined by lot, or it can be kept secret, revealed only through the course of voting. Let us consider first the case in which the complete sequence is revealed following its determination. Assume first that the voters have made proposals analogous to those in the two-person example $P(B - 2, 1, 1)$, P' $(1, B- 2, 1)$, P'' $(1, 1, B - 2)$, S, and that the order of voting is the proposer of the first issue, followed by the second, and then the third.

The advantage of voting first has now been greatly reduced. Voter 1's best strategy is to remove P', for if he does not, voter 2 will remove P, and P' will win. But in removing P' voter 1 assures himself of only a fifty-fifty chance that the second voter does not reject his proposal, voter 2 being indifferent between P and P''.

Voter 1 could ensure the final selection of his proposal if it contained higher benefits for voter 2 than 3's proposal. If voter 1 had proposed $P(B - 6, 3, 3)$ with P' and P'' as before, the second voter would be sure to pick P over P''. Thus, with 3 or more voters, the relative magnitudes of the payoffs to each voter are important as well as (indeed more so than) the order of voting.

To see this more clearly, assume the three proposals are

$$P_1(B - 6, 3, 3), \quad P_2(2, B- 4, 2), \quad P_3(1, 1, B - 2).$$

(The subscripts denote the rank order of the proposals based on the size of benefits to voters other than the proposer, a convention adopted again later.) There are six possible sequences in which the three voters can be drawn. Once the sequence is known, each voter can determine his best veto vote for bringing about either the selection of his own proposal, or if that is impossible, the selection of the other issue most favorable to him. Suppose, for example, the sequence of voting is 321. The best voter 3 can do is ensure the victory of 1's proposal. If voter 3 eliminated P_1, voter 2 would eliminate P_3, and P_2 would become the committee choice. The best 3 can do is eliminate P_2, leaving 2 to eliminate P_3, and P_1 to emerge as the committee decision. The greater attractiveness of P_1 to the other proposals results in its selection via the voting procedure in five of six possible orderings of the three voters. Only when the ordering 213 occurs will P_1 not become the committee decision, as voter 2 will eliminate P_1 and voter 1 P_3 leaving P_2 to emerge as the committee decision.

Thus, when there are three voters, the relative magnitudes of the benefits proposed for the other voters is more important than the order of voting. With two voters, a voter can ensure the victory of his issue when he votes first, by vetoing the other voter's proposal, and proposing an issue slightly better than the status quo. With three voters and the order of voting known, the least popular of the three proposals has no chance of winning, and the proposal with the highest benefits to other voters emerges as the committee decision five-sixths of the time.

This result does not hold for the alternative variant of the procedure in which the subsequent order of voting is not initially revealed to all voters. In general every proposal promising some positive benefits to all voters can have a positive chance of winning under this alternative to the procedure, although the proposal promising the most even benefits still has the highest probability of winning. Since discussion of this procedure is far more complicated, and the outcomes under it seem in every way inferior, we limit discussion throughout the rest of the essay to the variant in which the complete order of voting is announced before the veto voting stage begins.

2. The *n*-Person Case

Let us now examine the properties of the voting procedure more systematically for the general case. Assume there are n voters and therefore $n + 1$ proposals $P, \ldots, P^{n+1}$ including the status quo. Each proposal P^j is an n-element vector $p^j(x^j_1, x^j_2, \ldots x^j_n)$ designating payoffs to each of the n voters, where $B \geqq x^j_i \geqq 0$, for all i and j. At this point we shall place no constraints on the distribution of the x *within* any proposal. It simplifies the discussion without affecting the conclusions, however, to assume that each proposal promises a given voter a different payoff, i.e.,

$$x^j_i \neq x^k_i \text{ for all } i, \text{ and all } j \neq k.$$

This assumption allows us to determine a unique ordering of all proposals for each voter.

Obviously, in a choice between any proposal and the proposal he ranks last, a voter always eliminates the latter. Thus, W, the winning proposal, can never be the proposal ranked last out of the entire issue set by the voter who is last in the voting sequence. Call this proposal R_n.

By the same reasoning, W cannot be the proposal ranked last by the second last voter. Suppose the second last voter and the last voter rank the same issue last, then W cannot be the proposal ranked second last by the second last voter. The argument is similar to the above. The second last voter knows that R_n cannot win, since the last voter will always reject it. If the second voter is confronted with a trio of proposals containing R_{n-1}, the issue he ranks second last, he will reject it, since he must prefer whatever issue the last voter would leave from the remaining pair, R_n being certain to be eliminated. In this way we can work our way back through the n voters and associate a given issue R_i with each voter i in the sequence, which can be rejected as a possible winning proposal, and a given set of issues $E_i\{R_i, R_{i+1}, \ldots, R_n\}$, taken together, which can be eliminated as possible winning issues. The R_i and E_i for any voter i can be found by applying the following rule.

RULE 1. *Define* $E_{n+1} \equiv \{0\}$. *To find* R_i *for any* $i = 1, n$, *first remove the set* E_{i+1} *from the full set of* $n + 1$ *proposals.* R_i *is voter* i'*s lowest ranked proposal in the subset of remaining proposals,* $E_i \equiv \{R_i\} U E_{i+1}$.

THEOREM 1. W *cannot be* R_i *for* $i = 1, n$.

Proof is by induction. The proposition has been demonstrated above for the nth voter. Assume for the ith voter that no element of E_{i+1} can win. Let R_i be the lowest ranked proposal in the remaining set. Since no element in E_{i+1} can win, the eventual winner must be in this remaining set. If R_i would win, if voter i did not reject it, voter i will eliminate it from the set, since he prefers all other issues in the remaining set to R_i. Thus, R_i cannot win.

Now consider the decision by voter $i - 1$. We know from rule 1 that voter i prefers R_{i-1} to R_i. Thus, if both R_i and R_{i-1} are passed on to voter i, he will reject R_i, if it could win, leaving R_{i-1} to continue on as a possible winning issue. But voter $i - 1$ can ensure the victory of an issue he prefers to R_{i-1} by rejecting this issue. In the presence of R_i and the other issues in the initial set remaining after E_{i+1} is deleted, we know that voter i rejects R_i. Since W cannot be in E_{i+1}, it is in the set of remaining issues, all of which voter $i - 1$ prefers to R_{i-1}. Thus, voter $i - 1$ will always reject R_{i-1} if he believes it would win, and neither R_{i-1} nor R_i can win. But, since $E_i = \{R_i\} U E_{i+1}$, we have also shown that no element of E_i can be W.

E_1, the set of proposals eliminated as possible winners after considering the preference orderings of all n voters, is an n-element set. Since the entire set of proposals contains only $n + 1$ elements, we have the following theorem

THEOREM 2. *The voting procedure defines a unique winning proposal* W *for any set of initial proposals, and a given sequence of voting.*

Under the procedure each voter is effectively confronted with a choice between two proposals. The first voter must choose an issue to be eliminated from the entire set of $n + 1$ proposals. He knows, however, that $n - 1$ of these proposals, the E_2 set, cannot win even if all are left in the issue set passed on to the second voter. So the first voter's actual choice is between the two issues not in E_2. His best strategy is to reject the proposal he least prefers,

R_1, and pass the other along with E_2 on to the second voter. This voter knows that the $n - 2$ proposals in E_3 cannot win, and so must effectively choose between R_2 and the other proposal left in the choice set by voter 1. His best strategy is to reject R_2 and pass the remaining proposal, W, and E_3 on to the third voter. Eventually, the last voter is left with the choice between R_n and W.[5] Each voter can do no better than to eliminate the least preferred of the two proposals from which he has an effective choice. Honest voting at each step of the veto voting sequence is a Nash equilibrium.

To learn more of the properties of the winning proposal we must examine the incentives for formulating the proposals, in the issue proposal stage. The choices facing a committee member at this stage can be conveniently divided into the amount he assigns to himself, and the distribution of the remainder among the other $n - 1$ voters. It is obvious that a voter will never propose a less than proportionate division of the benefits for himself. If he did it would be possible that he preferred someone else's proposal to his own, and had to eliminate his own. But then he should have proposed a greater share for himself at the issue proposal stage. The decision at the issue proposal stage is thus the amount to assign to oneself above B/n, and the division of the remainder among the other voters.

An obvious strategy is to give each of the other voters an equal share, say x, of the benefits not assigned to oneself. Now consider proposing slightly more for some other voter i (say $x + a$), and slightly less for some other voter j $(x - a)$. Given the mechanics of the procedure, the proposal will be vetoed whenever it provides less benefits to a voter than the proposals remaining in this voter's effective choice set. Call these minimum amounts for voters i and j, r_i and r_j. An asymmetric division of the benefits improves a proposal's chances of winning only if equal division leads to i's rejection of the proposal $(x < r_i)$, while the asymmetric division does not $(x + a > r_i)$, and also does not induce j's rejection $(x - a > r_j)$. On the other hand, the proposal's chances of winning are lowered, when it would not have been rejected under the equal division strategy ($x > r_i$ and $x > r_j$), but is with an asymmetric division $(x - a < r_j)$. The probabilities that a proposal's chances of winning become higher (Π_H) or lower (Π_L) under asymmetric division of benefits are thus

$$\begin{aligned} \Pi_H &= \Pi\,(x < r_i < x + a) \cdot \Pi\,(x - a > r_j), \\ \Pi_L &= \Pi\,(x - a < r_j < x) \cdot \Pi\,(x > r_i). \end{aligned} \tag{1}$$

The proposer must assume that other proposers are going through calculations similar to his, and it would be reasonable for him to assume that the amounts offered on their proposals fall symmetrically around his. The probability that the amount offered a given voter on any other proposal falls in a small interval above the average amount he proposes should equal the probability the amount offered falls in the same sized interval below the average he proposes.[6] Thus, $\Pi\ (x < r_i < x + a) = \Pi\ (x - a < r_j < x)$. But, again given the symmetry inherent in the game, the expected values of r_i and r_j must be equal, and $\Pi\ (x > r_j)$ must be greater than $\Pi\ (x - a > r_j)$. Distributing a given quantity of benefits asymmetrically over the other voters lowers the probability of a proposal's success.

The $n + 1$ proposals can now be ordered on the basis of the amounts proposed, equally, for the $n - 1$ individuals other than the proposer. Let P_1 be the proposal offering the largest amounts to the other voters; P_{n+1} the smallest. If the status quo is a zero payoff for all voters, $P_{n+1} = S$. If not, we can, for simplicity, assume that it takes the form of the other proposals ($B/n + e$ goes to one voter and the rest is evenly divided), and include it in the ordering at its appropriate position.

Now let us see which of the proposals emerge as winners under the procedure. It is again useful to begin with the decision of the last voter. If he has not proposed P_{n+1}, he ranks it last and $R_n = P_{n+1}$. If the voter n has proposed P_{n+1}, he ranks P_n last and $R_n = P_n$. Thus, E_n must be either P_{n+1} or P_n. These possibilities along with those for the other $n - 1$ voters are summarized in table 1. Since either the nth or the $(n - 1)$th voter must *not* have proposed P_{n+1}, this proposal must be ranked last by one of them, and therefore must be an element of E_{n-1}. In addition, the E_{n-1} set must contain either P_n or P_{n-1} depending on whether the $n - 1$th voter has not or has proposed P_n. If we continue to examine the possibilities for each voter, we see that the size of the two possible elimination sets continues to expand by one, and the list of possible winners (the last column of table 1) to contract by one, until after considering all of the options for the first voter only two possible winning issues remain, P_1 and P_2. When any voter, other than the proposer of P_3 votes second, $E_2 = \{P_{n+1}, P_n, \ldots, P_3\}$. The first voter is effectively constrained to a choice between P_1 and P_2. If any voter other than the proposer of P_2 votes first, P_1 will be the committee choice. When the proposer of P_3 is second, the first to vote is limited to a choice between P_1 and P_3. Since the first to vote cannot be the proposer of P_3 in this case, P_1 must be the winner whenever P_3's proposer is second in the voting order. Thus, P_2 is the committee decision only when its proposer votes first, *and* he is not followed by the proposer of P_3. In all other cases P_1 wins.

As an alternative way of seeing why this must be so, notice that P_1 generally wins because it is the *second* choice of $n - 1$ of the voters, who work for its victory in the absence of a chance of their proposal's winning. When the proposer of P_2 goes first, however, he can put his proposal in this position by eliminating P_1. It pays him to do so, unless he is followed by the proposer of P_3. If he eliminated P_1 when followed by the proposer of P_3, the latter could make P_3 a winner by eliminating P_2. Thus, in this case, the proposer of P_2 also works for the victory of P_1.

If all voters have an equal probability of occupying any position in the voting sequence, the probability of P_2's winning, $\Pi(P_2)$, is the probability that he does vote first *and* is *not* followed by the proposer of P_3, i.e.,

TABLE 1

Voter	Issue Eliminated (R_i)	Possible E_i	Possible Winning Issues
1	P_3 or P_2 or P_1	$E_1 = \{P_{n+1}, P_n, \ldots, P_2\}$ or $\{P_{n+1}, P_n, \ldots, P_3, P_1\}$	$\{P_2, P_1\}$
2	P_4 or P_3 or P_2	$E_2 = \{P_{n+1}, P_n, \ldots, P_3\}$ or $\{P_{n+1}, P_n, \ldots, P_4, P_2\}$	$\{P_3, P_2, P_1\}$
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$n-2$	P_n or P_{n-1} or P_{n-2}	$E_{n-2} = \{P_{n+1}, P_n, P_{n-1}\}$ or $\{P_{n+1}, P_n, P_{n-2}\}$	$\{P_{n-1}, \ldots, P_1\}$
$n-1$	P_{n+1} or P_n or P_{n-1}	$E_{n-1} = \{P_{n+1}, P_n\}$ or $\{P_{n+1}, P_{n-1}\}$	$\{P_n, P_{n-1}, \ldots, P_1\}$
n	P_{n+1} or P_n	$E_n = \{P_{n+1}\}$ or $\{P_n\}$	$\{P_{n+1}, P_n, P_{n-1}, \ldots, P_1\}$

$$\Pi(P_2) = \frac{1}{n} - \frac{1}{n(n-1)} = \frac{n-2}{n(n-1)}. \tag{2}$$

$\Pi(P_2)$ converges on zero as n approaches infinity. Thus, when the order of voting in the second stage is known to all voters, the probability that the proposal promising the most even distribution of benefits to all voters is chosen approaches 1.0 as the size of the committee becomes large.

This result clearly illustrates the relationship between the committee's size, and the importance of the issue proposal step relative to the order of voting in the veto voting step. With two voters, the nature of the issues proposed only ensures that a Pareto-preferred outcome is obtained; the order of voting determines how the gains are distributed. With n infinitely large, the order of voting is irrelevant; the benefits from collective action are distributed according to the most egalitarian of the proposals made.

The next question to consider is how egalitarian this distribution can be expected to be. In making a proposal each voter must decide the level of benefits, e, above the average benefit to award to himself. If his proposal wins he receives $(B/n + e)$, and if it loses $(B/n - d/(n-1))$, where d is the benefits above B/n the proposer of the winning proposal awards to himself. If the committee meets but once to decide a division of a single B, it is reasonable to assume that a given committee member assumes that the proposals of the other committee members are independent of his own. The probability of his proposal winning, $\Pi(e)$, can be assumed to be a decreasing function of e. The expected payoff if he loses, d, can be assumed fixed, and the individual's problem is to choose an e to maximize his expected utility $E(U)$, where

$$E(U) = \Pi(e)U\left(\frac{B}{n} + e\right) + (1 - \Pi(e))U\left(\frac{B}{n} - \frac{d}{n-1}\right). \tag{3}$$

Under reasonable assumptions about U and Π a maximum exists for a nonnegative e.

More can be said about the choice of e if we assume that the same committee meets repeatedly to make similar collective decisions. Under such circumstances, an individual can no longer assume that the choice of e by other voters is independent of his own. Since the proposal with the lowest e (or occasionally the second lowest e) wins, the contest for choosing the winning e resembles a Bertrand oligopoly game in which all the sales and profits in the market go to the firm with the lowest price. As in a Bertrand oligopoly situation, one expects the winning e (lowest price) to converge on zero. As in the oligopoly game, however, there remain some potential gains from successfully proposing a nonzero e. But, unlike in the oligopoly game, there are also some risks since the proposal of an $e > 0$, carries with it the risk of a loss of $d/(n - 1)$, should some other proposal promising $d > 0$ win.

A voter contemplating a choice between a long-run strategy of proposing a series of nonzero e's around some $\bar{e}$, versus always proposing $e = 0$, can, given the symmetric nature of the game, reasonably assume that his proposal will win $1/n$ proportion of the time. Due to this symmetry, he can also expect the average e he proposes on those occasions he wins to equal the average e proposed by others when they win, i.e. that $\bar{e} = \bar{d}$. Thus, his expected long-run gains, G, from a strategy of proposing a series of nonzero e's is

$$G = \frac{1}{n}\left(\frac{B}{n} + \bar{e}\right) + \frac{n-1}{n}\left(\frac{B}{n} - \frac{d}{n-1}\right) = \frac{B}{n} + \frac{\bar{e} - d}{n} = \frac{B}{n}. \tag{4}$$

But, when n is large, the voter can virtually ensure himself a payoff of B/n by proposing an $e = 0$, and making his proposal at least as good as P_1. With n very large, the latter strategy has the same expected value, over an obviously smaller spread of outcomes. Thus a risk-averse voter should always propose an $e = 0$. With one risk-averse voter, the egalitarian distribution of benefits occurs with probability $1 - [(n-2)/n(n-1)]$. With two risk-averse voters, it always occurs.[7]

An alternative way of viewing the problem of proposing a division of B under the veto voting procedure is as an n-person, constant sum, noncooperative game. Under the rules of the procedure this game is symmetric and has a Nash equilibrium.[8] It is also easy to see that the strategy of proposing an equal distribution of benefits is a Nash equilibrium, i.e. if $n - 1$ players propose an equal division, the nth player can do no better by proposing any other division. Indeed, the equal division of benefits is far more "robust" than implied by the Nash equilibrium concept. The equal division outcome will occur as long as any two voters propose it, making their proposals P_1 and P_2, and is impervious to the nature of the proposals by the other voters, so long as the assumption of no coalitions is maintained. Thus, even when the game of dividing B is played only once, a choice of $e = 0$ by only two voters is sufficient to ensure an equal division of B as the committee decision. These considerations lend further plausibility to the likelihood of the equal division of the benefits from collective action being the outcome to either the one-time-only committee decision, or of the "super game" played over and over again by the same players for similar stakes.

3. Determining the Allocation of Public Goods by Veto Voting

It is easiest to begin applying the rule to decide public goods by assuming that only one dimension of the public good is to be decided. Let us assume, therefore, that the quantity and characteristics of the public good are predetermined, and the committee must decide the distribution of tax shares. Consideration is limited to tax proposals varying with some generally held and objectively measurable characteristic, e.g., income, consumption, property value. Each voter is able to calculate his own tax under every proposal, and those of the other $n - 1$ voters. Each can determine his own ordering of the n

$+ 1$ proposals, and those of all other voters. Theorem 2 holds, and a unique winning proposal exists for any set of proposals and order of voting.

Suppose that a property tax is being used, and that all tax proposals are restricted to a combination lump sum charge, and proportional rate, i.e. that $t_i^j = a_i + b_i V^j$, where t_i^j is the tax on voter j for tax proposal i, and V^j is the value of voter j's property. Each tax proposal is uniquely defined by the two parameters a_i and b_i. These parameters must be chosen to satisfy the constraint that the entire tax revenue equals the cost of the public good, C, i.e.

$$C = a_i n + b_i \sum_{j=1}^{n} V^j. \tag{5}$$

Higher values of a_i are accompanied by lower b_i and imply lower progressivity under the tax. If we assume that S takes the form of the other proposals and satisfies (5), a unique ordering of the $n + 1$ proposals on the basis of their progressivity exists. Given the constraint of equation 5, an individual's tax under a more progressive tax scheme is higher or lower depending on whether his property is valued at above or below the *mean* property value of the community. All individuals with property values above the mean will rank tax proposals from most regressive (highest a_i) to least regressive. All individuals with property values below the mean have the reverse ranking.

It is easy to show that the winning proposal is the $(r + 1)$th most progressive tax schedule (i.e. with the $[r + 1]$th lowest a_i), where r is the number of voters with property values above the mean. If the distribution of property values is symmetric around the mean, $r = n/2$ and W is the proposal with the median degree of progressivity (the median a_i). If the distribution is positively skewed, $r < n/2$ and the amount of progressivity of the winning proposal exceeds that of the median proposal. The r voters with property values above the mean can eliminate only the r most progressive proposals, and the next most progressive proposal wins, it being preferred by all other voters.

These results are important in appraising the normative properties of the outcomes under the procedure. With the quantity and other characteristics of the public good fixed, the determination of the tax shares to finance the public good is a zero sum game. Different proposals make some better off and some worse off and Pareto-preferred proposals are infeasible. Some proposals may contain tax shares exceeding some individuals' gross benefits from the public good. These seem most likely to be at the two tails of the distribution, the most and least progressive of the proposals. Their elimination under the procedure increases the likelihood that it yields outcomes Pareto-preferred to the status quo.

Now consider the other extreme possibility. A public good is to be provided *free*, and the committee must decide the amount to accept. Assume that each committee member has a symmetric single-peaked preference for the good, which attains a maximum at a finite quantity. If each individual proposes his most preferred quantity of the public good, it is possible for all voters to determine a unique ranking of the proposals for each other voter. The conditions of theorem 2 are again satisfied, and the proposal with the median quantity of public good wins. If voter preferences are not symmetric, some errors in predicting the other voters' rankings might occur, but the tendency for the procedure to yield the median outcome would remain.

Before turning to the more complicated public goods case, it is important to consider whether it is in the interests of a voter to propose his most preferred quantity of public good. For simplicity we abstract from the complications that can arise when the proposer of the second best proposal votes first and assume a single, winning issue x_M, the median quantity proposed, in a community of n. Now add voter i to the committee with preferred quantity x_i. Consider first the case where $x_i > x_{M+1}$, and x_{M+1} is the next highest quantity to x_M proposed. The rejectors of quantities above x_M are voters preferring quantities equal to or less than x_M. The addition of x_i to the issue set causes a rearrangement of the vetoes of the committee. The individual who previously rejected x_{i-1}, the next smallest quantity to x_i, in a choice between it and still smaller x_M, now rejects x_i. The voter who rejected x_{i-2} rejects x_{i-1}, and so on until the voter who rejected x_{M+1} rejects x_{M+2}, leaving x_{M+1} uneliminated. Voter i will reject some quantity less than x_M causing the previous vetoer of this quantity to veto a higher quantity, and so on until the previous vetoer of the quantity just smaller than x_M is left to veto x_M. The introduction of voter i with proposal $x_i > x_{M+1}$ results in a shift in the committee outcome from x_M to x_{M+1}.

Now suppose $x_M < x_i \leqq x_{M+1}$. Voters who rejected x_{M+1} and higher quantities in favor of smaller x_M will continue to do so in the presence of x_i which is no greater than any of these other quantities. Voter i can reject a quantity smaller than x_M causing a rearrangement of the vetoes in the direction of higher quantities, and the elimination of x_M, x_i becomes the new committee decision.

Thus, if all other voters propose their most preferred quantities, a proposal by voter i of his most preferred quantity x_i, either shifts the committee decision to the next closest proposal (x_{M+1} if $x_i > x_{M+1}$, and x_{M-1} if $x_i < x_{M-1}$), or results in x_i's selection itself (when $x_{M-1} \leqq x_i \leqq x_{M-1}$). When $x_i > x_{M+1}$ or $x_i < x_{M-1}$, no other proposal strategy can do better. When $x_{M-1} \leqq x_i \leqq x_{M+1}$, no other strategy is as good. The proposal of a voter's most preferred quantity is a Nash-equilibrium.[9]

Now let us turn to the decision to provide a public good along with a tax to finance it. If the provision of the public good is potentially Pareto-preferred to the status quo, then collective benefits from its provision exist, and the question is how to divide these benefits. The question resembles the problem of dividing B discussed above. A voter can assume that the smaller the benefits he proposes for any individual (i.e. the higher his tax share or the further the quantity is from his preferred quantity), the higher the probability that this individual rejects the proposal. In deciding how much public good to propose and the distribution of tax shares, an individual must speculate on the public good quantities and tax shares the other proposals contain, and each voter's evaluation of them. Thus, an issue proposer is forced to make interpersonal utility comparisons of the benefits to other voters his proposal contains versus the benefits he expects the other proposals to contain. In the absence of information regarding the content of the other proposals (implicit or explicit coalitions of the type described in section 4), the most reasonable strategy is again to treat each individual symmetrically. By analogy with the division of *B* example, an individual can expect to minimize the probability of his proposal's rejection by promising equal increments in benefits to all other voters. Any asymmetric distribution of benefits, unless matched by the other proposals, must raise the probability of the discriminated-against voter's rejecting the proposal by more than it reduces the probability of rejection by those favored under an asymmetric distribution. By further analogy with the division of *B* example, we can expect an individual to propose for himself the same increase in utility from the provision of the public good as he proposes for everyone else. If he proposes more for himself than for others, his benefits should his proposal win are higher, but the chances of his proposal's winning are lower. Given the symmetric character of the game, the expected value of his benefits is the same for either proposal, but the risk attached to proposing greater benefits for oneself is higher. A risk averse voter should propose a fair sharing of the benefits from collective action.

The issue of interpersonal utility comparisons has now arisen. But, if we have learned one thing from the work on Arrowian impossibility theorems, it is that the fundamental distribution issues inherent in collective decision making cannot be solved without introducing interpersonal utility comparisons (see, e.g., Sen 1970, 123–25). If the distributional question is to be faced at all, the issue is not *if* interpersonal comparisons will be made, but *how.* Voting by veto follows the essentially democratic procedure of having each committee member make his own set of interpersonal utility comparisons, and then selects that one which emerges as most egalitarian in the eyes of the other committee members.

The information requirements for application of the procedure in the general public-goods–tax-share case are obviously rather demanding. The procedure is more applicable in committees in which each member has a good deal of knowledge about the preferences of the other members than in committees in which all members are essentially strangers. Citizens at a New England town meeting might be expected to know a fair bit about the wealth and tastes of each other, for example. More generally, one could envisage the procedure operating in a parliamentary committee in which each member represented a separate group of voters. Knowledge of the preferences of each committee member would then come from knowledge of the representative's constituency, and the platform upon which he won election. Incentives to deceive other committee members as to what one's preferences are would be constrained by the need to win election and "answer to the voters."

Even in such settings the problem of proposing a public-good–tax-share combination providing equal utility increments to all voters is a good deal more difficult than that of proposing a division of cash B. Voters will err, proposing tax shares which are too high for some, given their evaluations of the public good, and too low for others. But here again, the mechanics of the procedure work to eliminate those proposals with the most discriminatory provisions. The emergence of the proposal incorporating the "median view" of the pattern of utility functions in the community can be expected.

4. Coalitions and Redistribution

Coalitions that form before the proposals are made can agree to lower the benefits assigned to specific other voters, and divide the extra benefits among themselves. To succeed, however, a coalition must be at least one larger than the group from which it attempts to redistribute to survive the veto votes of this group. It must also promise benefits to all committee members not in either the discriminated-against group or the coalition itself at least as high as the next best proposal. Thus when the size of coalitions that can effectively form is small relative to the committee's size, a roughly even distribution of benefits across all members can be expected, although some small groups might be discriminated against, and Pareto-preferred outcomes cannot be assured.

When more than half of the committee forms a coalition, there is no way to prevent it from redistributing income away from the other members. In this regard, the procedure resembles majority rule, and a significant difference exists between the outcomes obtainable under this procedure, and the rule of unanimity. Under the unanimity rule each voter can be certain of protecting his interests in the face of coalitions of any size through his use of the unlimited veto the voting procedure gives him.

In addition to formal coalitions against specific groups, the appearance of tacit, informal coalitions against certain minorities is possible. An ethnic or religious minority may have traditionally received a smaller share of the benefits from collective action. Assigning this minority a less than equal share of these benefits may constitute a form of "Schelling point" proposal that many committee members automatically make. The outcomes under voting by veto are thus dependent on both the views of each voter as to what the other voter's preferences are, and their expectations of the form the other proposals will take. Equal division of B occurred as the likely equilibrium in the first example because no 'consensus' was assumed over a possible deviation from an equal division.

An implicit assumption underlying the arguments in favor of the voting procedure is, thus, that a basic value consensus exists among the committee members on the underlying distribution of income, property rights, and general scope of government activity. The view here is essentially Wicksellian.[10] Given general consensus on these basic issues, each individual enters into collective activity, as into voluntary exchange, to increase his own welfare. Each acts independently and treats all others individually and symmetrically while taking part in the collective choice process. No coalitions, either explicit or implicit, of one *group against another* exist.

5. Summary and Conclusions

Under the usual assumptions made about voting procedures, committee members are not charged with responsibility for proposing issues. Each simply reveals his preferences, honestly or otherwise, via a yes or no vote on the issues that somehow come before the committee. Under the unanimity rule this structure gives each individual an infinite number of vetoes against the issues proposed, raising the spectre of an endless rejection of issues as some voters attempt to increase their shares of the benefits at the expense of others. When less than unanimous agreement can decide an issue, outcomes leaving some worse off than under the status quo can occur. What is more, it is likely that no proposal is capable of obtaining the required majority against all others. The spectre of an endless chain of defeated issues again appears.

The voting rule described in this paper charges each voter with the responsibility for making one proposal, and limits him to a single veto over another issue. Nevertheless, the vetoes the other voters hold force the voter to consider and attempt to define the form of collective decision they prefer, both with respect to the status quo and the other proposals, while at the same time revealing the form of collective decision he prefers. Instead of being able to conceal his own preferences and ignore those of others, each voter is forced into a kind of proposal popularity contest in which he must seek out the preferences of the other voters and weigh their interests against his. The endless defeat of each issue is avoided by forcing a choice from among the n proposals the committee members make and the status quo.

The probability of the status quo emerging as the committee outcome under the unanimity rule looms large owing to the unlimited number of vetoes each individual possesses. In contrast, the status quo appears as "just another issue" under the present procedure, certain to be rejected if the issue to be decided is capable of yielding a Pareto-preferred outcome. In this way, the procedure resembles more closely a simple plurality rule. Its outcomes also resemble those of a plurality rule when the proposals are capable of ordering along a single dimension like the degree of progressivity, or the amount of public good. When this can be done, the committee decision is drawn toward the mean or median outcome as under majority rule. Last of all, the outcomes under the procedure can resemble those under majority rule whenever tacit or formal coalitions form a consensus over some attribute of the issue to be decided, like the degree of progressivity of a tax. An individual or small minority, which finds its views out of tune with those of the majority on a given aspect of the issue is unable to prevent the majority from having its way. In situations like this, non–Pareto-preferred outcomes are possible; redistribution of this type can take place, and outcomes from the procedure again resemble those under majority rule.[11]

Thus, the voting by veto procedure combines some of the best, to my mind, attributes of both the unanimity and majority rule. When no consensus among a majority of voters exists, and no coalitions are allowed, it should tend to produce Pareto-preferred outcomes with an equitable sharing of the benefits from collective action. The right of veto each individual possesses offers some protection against an indiscriminate reduction in welfare through a proposal, which singles him out for ill-treatment. Yet the minority cannot indefinitely block a majority which is in consensus on some basic attribute(s) of the issue. These features of the voting procedure seem to warrant further study of its potential as a democratic procedure for revealing individual preferences on public goods.

NOTES

Valuable suggestions for improvements were received from Steven Slutsky and a referee. Special thanks are due to my colleague Murat Sertel with whom I discussed the paper several times.

1. See Barry 1965, 242–50; Black 1958, 146–47; Buchanan and Tullock 1962, chap. 6; Rae 1975; and Samuelson 1969.
2. The literature on cycling is admirably reviewed by Amartya Sen.
3. This conclusion rests on the absence of coalitions among voters' assumption in cases where there are more than two voters (see sec. 4). On Pareto-efficient redistribution see Hochman and Rogers 1969.
4. Of course, each voter will want to offer the other the *minimum* amount above 0, and an existence problem exists unless B is something, like money, which is only finitely divisible.
5. It is not always necessary for the procedure actually to proceed by each voter eliminating the proposal associated with him by the rule. If voter $(i + j)$ would eliminate R_i, should voter i eliminate R_{i+j}, voter i is indifferent between the two actions. What is true is that the outcome from any alternative sequence of voting is *as if* each voter had eliminated the proposal assigned to him by the rule.
6. If anything this assumption is conservative. To win, a proposal must avoid being vetoed through each round of voting. On average the benefits promised to the other voters must be close to the highest of those proposed. A voter who thinks his proposal stands more than an even chance of winning must assume the other proposals' benefits are clustered below his, i.e., $\Pi\ (x - a < r_j < x) > \Pi\ (x < r_i < x + a)$ and the conclusion reached in the text, that $\Pi_L > \Pi_H$, is strengthened.
7. See Rothschild and Stiglitz 1970. With n very large, and $\Pi\ (P_2) \approx 0$, any degree of risk aversion is sufficient to produce a choice of $e = 0$. With small n, a given degree of risk aversion is necessary before the result holds.
8. Friedman (1971) proves the existence of a Nash-equilibrium for a game of this type defined under 3 conditions, all of which are satisfied by our voting by veto procedure.
   1. The strategy space of each player is compact and convex.
   2. His payoff functions are continuous and bounded on the strategy space.
   3. His payoff function, given a set of strategies by all other voters, is a quasi-concave function of his own strategy.

9. This conclusion is unaffected when the special cases dependent on the ordering of voters are considered. If the proposer of x_M votes first, he is in the position of the proposer of P_2 in the example considered above; P_1 is either x_j (if $x_i \leqq x_{M+1}$) or x_{M+1} (if $x_i > x_{M+1}$). The proposer of x_M can maintain the victory of x_M in the absence of x_i, by vetoing either x_i or x_{M+1} depending on $x_i \gtreqless x_{M+1}$. When this occurs, i is simply unable to improve the committee outcome no matter what he proposes. On the other hand, when x_i falls between x_{M+1} and x_{M+2}, x_i takes the form of P_2, and its proposer can bring about x_i's victory by vetoing $x_{M+1}(P_1)$, should he vote first. Thus voter i remains either better off, or no worse off, than under any alternative strategy if he chooses to propose his true, most preferred quantity.
10. See Buchanan's (1949) discussion of the Wicksellian approach, as well as Wicksell (1896).
11. Readers familiar with the Vickrey-Clark-Groves procedure for revealing preferences for public goods will also recognize parallels with the present rule (see, e.g., Groves and Loeb 1975). Most importantly, both rules charge the voter with revealing more information about his preferences than is contained in a yes-no vote, and build in incentives to ensure honest revelation of preferences. On the margin, the proposal (demand schedule) of a single voter is effective in determining the exact nature of the final outcome under both procedures, the other proposals having essentially cancelled each other out. But, the process of cancelling (adding up) is such that with large numbers of voters, the elimination of this, or any, voter does not significantly affect the outcome. There are other parallels, but we cannot pursue them here.

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