CHAPTER 6

Nonlinear Politics

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That chaos may be a part of elementary politics is evidenced by highly exploratory work in the fields of electoral behavior, game theory, axiomatic choice theory, and conflict analysis. The social dynamical processes that may induce chaos, methods of investigating large-scale collective behavior, and some implications for political science research are outlined in this essay.

Deterministic Chaos

At the turn of the century, Poincaré, and later Birkoff, noted that fully deterministic dynamics do not necessarily provide explicit predictions on the evolution of a dynamical system (Poincaré 1892; Birkhoff 1927). The subtlety of their observations was not fully appreciated until much later (Cartwright and Littlewood 1945; Levinson 1949; Smale 1967; Ruelle and Takens 1971). Today, chaos is a component in understanding dynamic macrobehavior in many sciences.

Chaos exists when the long-term prediction of a system is impossible because uncertainty in a system's initial state grows exponentially fast over time. Chaos fits the criterion that the autocorrelation function of the time signal goes to zero in finite time. Because trajectories are unstable, errors of estimation of initial conditions or parameters, however small, can later accumulate into substantial errors. This implies that forecasts of future behavior based on the past become problematic as current memory of the past fails. Nonlinearity is a necessary but not sufficient condition for the generation of chaotic motion. Of course this implies some feedback mechanism exists. The observed chaotic behavior is due neither to external noise nor to an infinite number of degrees of freedom. The source of irregularity is the nonlinear system's property of separating initially close trajectories exponentially fast.

Because the time evolution is self-independent from its own past history, predicting the long-term behavior of chaotic systems is an interesting exercise. The process does not come to rest in a stable equilibrium but instead

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comes to occupy large patches of state space. Analysis is difficult because stable and unstable states are strewn together in extremely complicated ways. The critical feedback process central to the dynamics under investigation therefore can be docile or unruly, depending solely on the tuning parameter values of the equation(s) of motion. Indeed, for all practical purposes the distinction between determinism and nondeterminism disappears. That is, even equations that are solely deterministic become nondeterministic in their long-term realizations.

One may assume it impossible to predict the long-term behavior of political systems that are potentially chaotic because their initial conditions can be fixed only with finite accuracy and subsequent error increases exponentially fast. For politics, this means political actors within a chaotic environment who are governed by the same laws and are identical in every measurable way may evolve differently over a long time. Again, this would occur despite identical initial configurations.

But this unpredictability of the long-time behavior exists only at the level of individual trajectories. At the level of statistical properties of the time evolution (averaged over different trajectories, say as they evolve from different nearby initial conditions), very definite predictions are possible. Trajectories will eventually move only on a small submanifold (chaotic or strange attractor) of the entire state space, with predictable visitation frequencies of the different parts of the attractor. It may be possible to estimate the time up to a chaotic event, offering a short-term prognosis of a political actor's trajectory.

So far the theory of chaotic dynamics has uncovered three major routes for the onset of chaos as the external system parameters are changed: the Ruelle-Takens scenario via quasiperiodicity, Feigenbaum's period doubling, and Pomeau-Manneville's intermittency route. Each has its own characteristic signature (e.g., in the autocorrelation function). Such signatures may provide important indicators for incipient changes. Other indicators may come from the fact that, generically, a system develops characteristic large-amplitude fluctuations as it approaches a phase transition. Within measured turbulent flows (mostly found in the physical and life sciences), chaos is known to exist. Thus, for any theory of dynamical politics to be viable, it must come to grips with political turbulence and hence incorporate chaos.

Gaps in our understanding of political dynamics have become particularly obvious and acute during recent political events in Europe, Russia, Japan, and the Middle East. These events and rapid macropolitical changes came as unexpectedly to any political scientist as the 1989 San Francisco earthquake came unpredicted by any geologist, or the 1993 rains in the Midwest by any meteorologist.

It is not that developments such as those in Russia or Eastern Europe could have been predicted in detail if better models had been available; the point is that *despite* the unpredictability of such developments, there may be definite patterns in their occurrence. Furthermore, there may be well-defined relations between the time scales on which short-term prediction is possible and the characteristics of chaotic evolution at long times; there may be indicators of incipient macropolitical changes; and there may be a wealth of macroscopic variables that are predictable at long times even though the system state is not. In the field of nonlinear dynamic systems, such coexistence of predictable and unpredictable properties has led to phrases like *deterministic chaos* (Schuster 1988) and *order in chaos* (Prigogine and Stengers 1984).

A common example is weather forecasting. Here the solutions of the relevant (Lorenz) equations of motion are so sensitive to small changes in initial conditions that predictions beyond two or three days are intrinsically impossible. In some places, like Los Angeles, the weather is pretty much the same from day to day. On those days, the utterance of any local weather person will be as correct in predicting the weather as forecasts of the entire U.S. National Weather Bureau. An inversion layer produces smog; the endless summer continues. Then one day the predictions break down. The massive computers around the world miss the formation of a Pacific storm that flattens the coast of Baja, California. At the same time, it is possible to predict average temperature, amount of rainfall, and so on, for the month of November in the year 2022 at any desired location for any given set of external parameters. The equations that govern weather can simultaneously behave both ways; order shifts to disorder and back through the mathematical passages of chaos.

Nonlinear Political Interactions

Politics, whether at the communal, state, national, or international level, results from the interactions of individuals. These individuals are members of respective political collectives; while some may be equipped with more power and attain an elite status, even the elites are interdependent. Political leaders base their decisions on interactions with their advisors; they form impressions of their advisors, who form impressions of them. Relationships change as these impressions and experiences inform subsequent interactions. Legislators chat in the halls before key votes and discuss issues with staff, advisors, and adversaries. At times they also check with constituents back home. What legislators say and do influences their standing in a legislative assembly and in their districts. Their standing in each one influences the other. The process is reflected down to the level of the typical voter who decides, say, to support a Republican or a Democratic candidate in an election, based on interactions with friends, family, or acquaintances in her local area. Even the individual voter's thoughts and actions may be the product of subtle self-interactions, as in Elster's multiple self.

A useful event that illustrates the impact of social dynamics *might* be Black Monday: October 19, 1987. *Might* is not a trivial adverb: the most widely accepted cause of the market's collapse, programmed trading, has been shown to have only a weak effect on market volatility (Grossman 1987), and as a consequence we really don't know what happened.

Brock (1991) interprets the market collapse as being the product of a herd behavior induced by the attenuation of normal information channels available to traders. His parable goes as follows: In a market functioning normally, traders regularly upgrade their information by calling on industry and security analysts. Communication between the traders and industry analysts is open. Prices are a function of both past price performance and volume, as well as expectations for future earnings and dividends. When a negative shock hits the market, prices drop. Traders use the behavior of other traders, price, and information from analysts to decide their next move. If the negative shock is large enough, the information channels between traders and analysts jam—too many phone calls. A trader is left with simply following the herd; the only existing information is the movement of prices and the observed behavior of other traders. As Brock suggests, a sell-off gets started when a positive feedback loop emerges in an otherwise insipid market. The individual traders' reward structures are likely to magnify this process into a serious collapse (LeBaron 1989).

Banerjee (1992) further argues that herd behavior can occur where a decision maker pays more heed to information others are perceived to have. The reason is simple: the visible actions of others may be based on information that the decision maker lacks. Private information is overridden by others' signals and the herd takes off. The event of an economic landslide suggests even catastrophic change, let alone small shifts in preference, can result from group interactive effects.

Unfortunately, attempts to analyze complex political behavior often tend to attribute observable cooperation or conflict to the individual. Behavior in politically intricate situations is attributed to higher levels of political information, a deeper understanding of the game's rules, or superior strategies, and of course lots of information (or at least enough information so as not to be duped). Nearly all published theory on the iterative prisoner's dilemma fits into this category. What must be remembered and used to calculate strategies includes: discount values, prior encounters, likely future encounters and associated probabilities, who might use what piece of information to figure out what to do, the cost of keeping information, and so on. Of course, nobody thinks this way. And even if anyone did, there is no guarantee that a success-

L. Douglas Kiel and Euel Elliott, Editors
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ful strategy at one time will not be utterly unsuccessful at another. In real political interactions are we left to laws of chance?

For decades we have seen glimpses of the complex individual-group nexus that defines politics. For instance, we know political beliefs are often based on socially cued ideologies by proxy. That is, individuals may faithfully reflect and follow over time the beliefs of others who possess developed abstract formulations of politics (Campbell et al., 1960; Converse 1964). That most voters lack much useful political information or fail to behave rationally is rarely debated anymore (Ferejohn and Kuklinski 1990). But the end of such debates doesn't mean we really understand the mechanism that induces even the most basic forms of political behavior. Turnout is one example. Rational choice theorists still cannot understand why anyone would vote, given the costs relative to proportional influence. Empirical electoral analysts cannot figure out precisely why aggregate voting rates appear to be going down—or are they going up this year? As a good friend in industry says, there is something wrong with this picture.

Social dynamics result from sets of local interactions between group members and their interactions with the environment. Logically, such signaling fits within a variety of classical social psychological perspectives that suggest individuals adapt to social environment (Asch 1951; Festinger 1957) or form impressions about others based on behavioral experiences (Heider 1958). In politics we know that decision makers modify their own behavior and the influence of social environments through self-selection (Schelling 1978; Finifter 1981), avoidance (MacKuen 1990), migration (Brown 1988), or a generalized contagion process (Huckfeldt and Sprague 1987, 1988, 1993).

Formally treating interactive political behavior within massively diverse collectives is tricky. Interactive behavior is peculiar in that it can neither be predicted nor analyzed by observing sets of individuals cross-sectionally, or even the time series from a given individual or group. Social dynamics and the concomitant social behavior cannot be reduced to individual behavior in the sense that isolated individuals cannot induce the variety and richness of global collective behavior prevalent in any political system. Social and political behavior is by definition holistic and synergetic (Haken 1978, 1983) and must be the product of interacting individuals who can communicate and modify their behavior as a consequence of their interactions.

Any time series is a rough statistical characterization of a collective process. A power spectral analysis alone, for instance, cannot decide the dynamical rules that model the time series. Both a deterministic logistic map with maximum parameter values (Mayer-Kress and Haken 1981) and a stochastic system can generate a white noise signal such that a flat spectrum cannot tell which model is correct. Figuring out what drives the show for

L. Douglas Kiel and Euel Elliott, Editors
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interactive social behavior poses some interesting problems. In political dynamics there are likely to be spatial and temporal phase transitions. For instance, we know that transition from a pattern selection phase to fully developed turbulence occurs via the intermittency (Kaneko 1989). Nonlinear interactions would almost demand that abrupt and widespread events occur unexpectedly. How to proceed is the question.

Among the possible ways to investigate dynamical politics are simulations of intriguing spatial arrays with cellular automata. Cellular automata are formal dynamical systems with many discrete degrees of freedom. The beauty of automata is that while the rules of interaction are surprisingly simple, complex nonlinearity can be induced by the iterative nature of interaction. As nonlinear systems, cellular automata can display the full array of dynamics of any real, living system, from fixed points to cyclical behavior to chaos. Can political life be the product of simple, primitive models? Cell-space models have been used in physics (Herrmann 1992), chemistry, and biology (Gutowitz 1991; Forrest 1991), and to a more limited degree in the economic and social sciences to investigate the ecological structure of behavior (von Hayek 1937; Schelling 1971, 1978; Cowen and Miller 1990). In politics, they may represent the potential to "program" the information available and used by entities (voters, groups, elites, nations, or whatever), and hence what we can discern from cellular automata may give us further insight into the information dynamics in real politics.

Rules can reflect essential existential states of being: love and hate. Love attracts and induces assimilation; hate repels and induces egestion. Rules can be deterministic or probabilistic. In both cases, cellular automata are formed on a lattice. Cellular automata can clearly produce patterns in space and time that can be moving fractal clusters if the underlying dynamics are chaotic. We can determine if the underlying dynamics are chaotic by watching the spread of small changes in the configuration of the system on a lattice.

A lattice is usually a d-dimensional array of discrete cells whose behavior is governed by local, uniform, and time-independent rules. Boundary conditions may be periodic. The rules determine the behavior of individual cells as they interact with others at the next time step. Future states of a cell are normally a function of the values of neighboring cells and the cell itself. The radius of a cellular automata is the number of neighbors whose state at time t affects the state of any cell at $t + 1$. The state of the cell is a variable, s_i, (where $i = 1, 2, \ldots, N$), which can be any measurable political trait. If $s_i = 0$ we can say the voter is a Democrat; if $s_i = 1$ the voter is a Republican. The state of the entire system, its configuration, is defined by the collection of values $(s_1, s_2, \ldots, s_N)$. From these values, the average fraction of partisans at any time step, $t, f_p(t) = ((1/N) \Sigma_i s_i)$, its variance, and so on, can be easily calculated, as well as the asymptotic fraction of partisans based on a long period.

Although individuals remain in fixed physical locations, their partisanship may change as a result of interactions with other voters on the lattice. This is a primordial political world, but well reflects many of the central ideas in contextual models of politics. By using political rules on a lattice, enough interactions between decision makers can replicate real political situations. Finding useful rules that map different forms of political behavior is the key to making automata theory viable for the social sciences.

A simple two-dimensional interaction law in which each individual interacts in the same way with its neighbors is the deterministic majority-rule law. This law makes each site adopt the value prevailing among the cell and its four nearest neighbors on the lattice. The interaction rule for such electoral behavior is given by

$$G_i(s_i(t+1), \ldots, s_N(t+1)) = \begin{cases} 1 & \text{if} \quad \sum\limits_{|i-j| \leq a} s_j(t) \geq 3, \\ 0 & \text{else} \end{cases}$$

where G_i are given functions of time t and the current values $s_1(t), \ldots, s_N(t)$; $|i - j| = \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2}$ is the Euclidean distance between cells i and j, with location (x_i, y_i) and (x_j, y_j), respectively. The summation is over all cells j whose distance from i is less than or equal to a, including s_i and its four nearest neighbors (fig. 6.1). Thus, the voter at cell i will be Republican at time $t + 1$ if, among i and its four nearest neighbors, at least three are Republicans at time t; otherwise, it will be Democratic. Since the summation condition

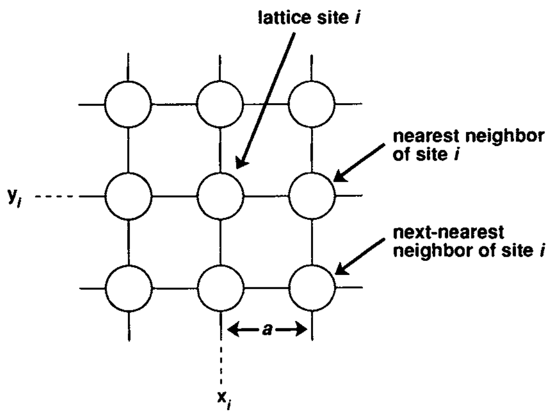

Fig. 6.1. A representative lattice site, *i*, and its neighbors on a square lattice with $D = 2$ and lattice constant *a*. (The coordinates x_i and y_i designate the physical location of site *i*.)

always examines five cells, no ties can occur and the interaction law is symmetrical with respect to partisan influence.

To see how the rule works consider i, starting from some given initial state, 0, its four nearest neighbors, and their respective nearest neighbors, over two time periods, as shown in figure 6.2. In the first time step, the interaction of the four neighbors of i with the peripheral cells leads to a conversion of the four neighbors into Republicans, leaving i Democratic. In the second time step, the central cell i also becomes Republican. As this simple illustration shows, over time a voter is affected by voters outside the range of the defined radius of the interaction law. Had we chosen the voter to the Northwest of i to be Democratic instead of Republican at $t = 0$, then i would have remained Democratic at $t = 2$. A more general statement would be that the state $s_i(t)$ will be influenced by the initial states $s_j(0)$ of all cells j for which the inequalities

$$\begin{aligned} |y_j - y_i + (x_j - x_i)| &\leq ta \\ |y_i - (x_j - x_i)| &\leq ta \end{aligned}$$

hold ($t = 0, 1, 2, \ldots$). Despite the simplicity of this political interaction rule and its explicitly local nature, it shows that the long-time behavior of the voter depends on the initial configuration of the whole system. A more complicated rule, or one with global characteristics, would be at least as sensitive.

Brown and McBurnett (1992) have used a stochastic version of the

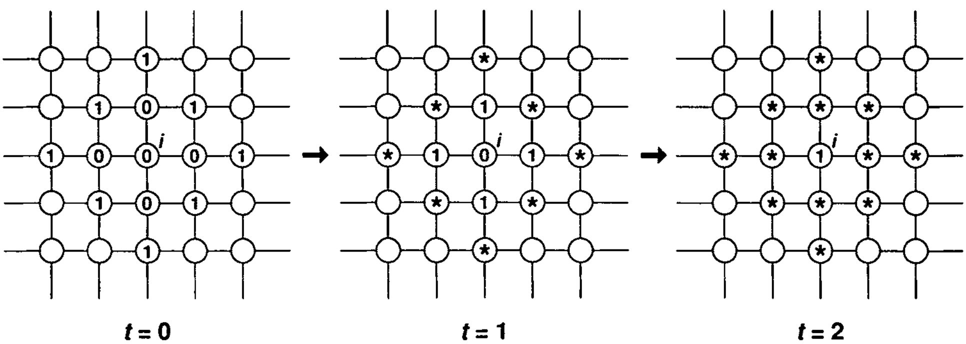

Fig. 6.2. First two steps of the time evolution of a restricted segment of the lattice, centered at some site *i*, under the deterministic majority law. The values in the circles display the partisanship of the different sites; the symbol * means that the state cannot be updated without additional knowledge of the initial configuration beyond what is shown for *t* = 0.

L. Douglas Kiel and Euel Elliott, Editors
http://www.press.umich.edu/titleDetailDesc.do?id=14623
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above model to simulate multiple political realities. In the model two opposing groups are arrayed on a 128 x 128 periodic two-dimensional lattice of initially randomly distributed sites. From such stochastic rules it is known that highly ordered systems evolve from what are essentially random configurations (Clifford and Sudbury 1973), though the specific evolved configurations are themselves apparently random and are highly sensitive to initial conditions (Durrett 1988). When a stochastic voting rule is applied to a random initial configuration, the system behaves in a fashion that is completely dependent on the initial concentration of Democrats or Republicans. Small chance events early in the history of a political system (or a nested subsystem) may fundamentally alter the numerical power balance between or among political actors.

While the asymptotic state of nearly all stochastic models such as that described above is unanimity, the finite time path provides insights into interesting political dynamics (Brown and McBurnett 1992; McBurnett and Brown 1992). For instance, a natural clustering of like voters emerges as an outgrowth of the interaction rule. Over time these clusters become a randomly built political landscape. (A time series describing this process is given in fig. 6.3.)

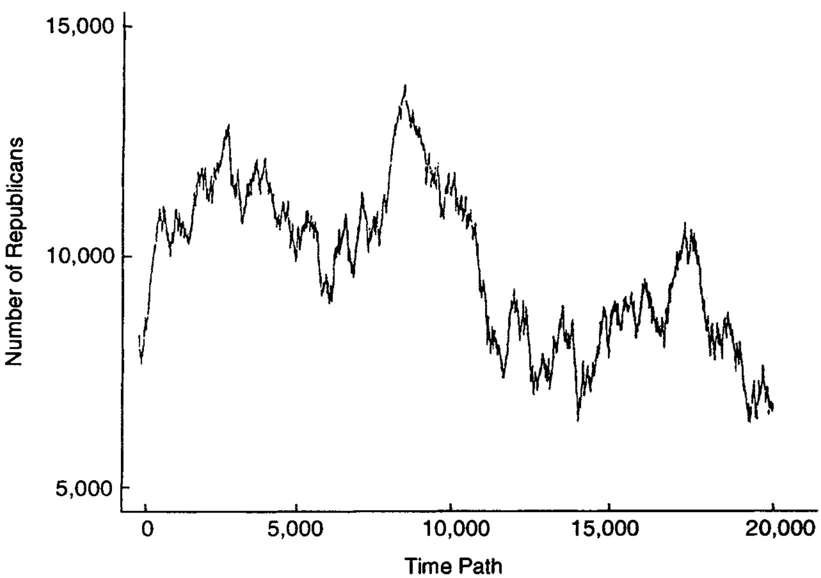

Fig. 6.3. Time series of the number of Republican voters governed by a stochastic voter rule as simulated on a 124 × 124 square lattice with a periodic boundary. Initial conditions are random. Time steps 1 to 20,000 are shown (Brown and McBurnett 1992).

L. Douglas Kiel and Euel Elliott, Editors
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The steady gain of one group comes at the expense of the second. The outcome of which group will dominate, however, is by no means predictable. At any point the fortunes of a leading group can be reversed. An equilibrium is not predictable in advance, in that the trajectory of the individual voter exists within a highly irregular political environment. The time series from these and other experimental data appears to have a fractal, noninteger dimension, though more experiments are needed to determine if this is true (Brown, Pfeifer, and McBurnett 1992; McBurnett and Krassa 1991).

Wolfram (1986) has offered the most complete classification of cellular automata. Cellular automata appear to be classifiable into four general types of behavior: Class I: cellular automata evolve into a fixed, homogeneous state and these structures evolve to fixed limit points; Class II: evolution to a simple periodic structure represented by limit cycles; Class III: chaotic, aperiodic patterns as found when strange attractors are present; Class IV: complex localized organizations as represented by systems that exhibit very long transients.

Distinguishable classes of cellular automata related to politics can only begin to tell us what the subtle, underlying structure might be. For instance, electoral systems may represent an example of Class IV behavior by exhibiting very long transients, a great deal of local order, and a global order that may emerge. Bureaucracies, on the other hand, may be reflected by Class I dynamics where both local and global scales of order exist. Yet the statistical behavior of such systems would still be unknown. Simple classification of dynamics is not sufficient to give us greater insight into political and social dynamics. While lists of political systems (and problems) classified by dynamical behavior would be an improvement over current methods, these classifications could not tell us precisely how the reality, process, and ordering of numerous forms of qualitatively differing systems came about or, more importantly, how they will evolve.

If political dynamics are likely to be complex or chaotic, how would we know? One set of measures would be based on the properties of the aggregate time series of the number of partisans, as well as on their asymptotic fraction. The correlation integral, the Lyapunov exponents, and spectral analysis are now commonly used to evaluate the presence of chaos (Peitgen, Jürgens, and Saupe 1992). On a spatial lattice, for the trajectories to be chaotic, slight perturbations in the initial conditions will have to induce a changed configuration in the values of cells. How two trajectories quickly become different is not the issue because the exact rule of interaction is known. We must instead estimate how rapidly and how far the trajectories will diverge. The notion of chaos in politics may indeed include the speed with which trajectories separate. In order for the concept of closeness of trajectories to have meaning, a measure of just what fraction of the voters are different for the initial and

L. Douglas Kiel and Euel Elliott, Editors
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The University of Michigan Press, 1996

perturbed configurations is needed (Herrmann 1992). The Hamming distance measure provides a useful definition of distance in phase space

$$\Delta(t) = \frac{1}{2N} \sum_i |\sigma_i(t) - \rho_i(t)|,$$

where $\sigma_i(t)$ and $\rho_i(t)$ are two time dependent configurations in phase space and N, as mentioned above, is the number of cells indexed by i. The dynamical behavior would be chaotic when in the thermodynamic limit $\Delta(t)$ gets big for large times if $\Delta(0) \rightarrow 0$.

While the work of Brown and McBurnett suggests that even simple political interaction models show a sensitivity to initial conditions, much more analysis is needed. To determine an exact dynamical behavior requires using in a simulation the same sequence of random numbers applied to two configurations of a lattice. Meaningful outcomes can only be determined after averaging over many initial configurations of equal distance and across different sequences of random numbers. Of course, in deterministic systems, once a set of configurations is identical they will stay identical. Once knowledge of the behavior of cellular automata models is gained, analysts will be in position to calibrate lattice rules to match both time signatures and spatial movements that match some categories of interesting political mass behavior.

Strategic Interaction

Models of social interaction investigated by means of cellular automata may open up for political science issues as yet unapproachable. We know that within strategies for solving common collective choice problems, cooperation (or conflict) may appear to be stable or even frozen in place while it can also instantly disappear with the slightest nudge or noise. Cellular automata can model conditions where a large number of actors make many decisions, as in any complex social community where individuals interact with imperfect information. However, knowing the rules that are important to investigate requires an adept touch at this point, because within the structure of automata theory, the number of k-state, r-radius rules is rather large ($k^{k^{2r+1}}$). It is probably best to look to familiar ground.

Determining how levels of cooperation or conflict are reached and spatially diffuse is a core element in iterative prisoner's dilemma research (Rapaport and Chamma 1965; Axelrod and Hamilton 1981; Axelrod 1984). As is well known, the prisoner's dilemma is a two-person, zero-sum game that can model the political evolution of either cooperation or conflict. Each player on every round can choose between cooperation (C) or defection (D), and the outcome is determined by a standard payoff matrix, V:

$$\begin{array}{ccc} & \mathrm{C} & \mathrm{D} \\ \mathrm{C} & R,R & S,T \\ \mathrm{D} & T,S & P,P \end{array}$$

where $T > R > P > S$ and $2R > T + S$. In a one-shot play of the game, each player's best strategy is to defect, since D gets the largest payoff, T, while C would get S. The paradox is obvious. In finitely many repeated plays of the game, all Cournot-Nash (noncooperative) equilibria have the property that no matter how long the sequence, a noncooperative outcome occurs in each period. In games of an indeterminate duration, however, the player's utility is calculated on the expected long-run payoff of repeated one-shot plays. In essence, it is best to figure out how to cooperate, since if both players use D both end up with a lower payoff since $R > P$.

Axelrod explored some spatial properties of the game with an eye on the territorial stability of tit-for-tat (Axelrod 1984). When the iterative prisoner's dilemma was examined within difficult environments, Richards (1990) showed the clear potential for chaotic behavior. In exploring evolutionary games played with neighbors on a spatial lattice, the most common approach is to set each lattice cell to a single strategy. The strategy may be either fixed (Axelrod 1984) or allowed to evolve based on a genetic algorithm or other optimization hill-climbing routine (Axelrod 1987; Miller 1989).

As before, the lattice mechanism is quite simple. All cells are updated simultaneously; the score of a cell is determined to be the average it obtains playing the neighbors defined by the given radius; the score of the cell is then compared to the scores of the neighborhood and the cell itself; the cell adopts the strategy of the highest score within the sites defined by the radius. Ties, when they occur, are usually broken by adding a touch of noise to the scores. As was observed above, the scores of the defined neighbors are dependent on the strategies of their defined neighbors, and so on. Thus, the contextual influence on any cell extends beyond the defined radius. When the model is restricted to set strategies, it clearly qualifies as a cellular automata.

To illustrate, let's examine one set of recent findings on the iterative prisoner's dilemma. In a paper, Nowak and May (1992) use fixed strategies of cooperation (C) and defection (D) to study how cooperative behavior evolves and is maintained. As mentioned above, nearly all prior work demanded that individuals or groups remember past encounters, with some discounting of retrospective evaluation for prospective payoffs. Such recollections are necessary because of the highly complicated strategies employed. Using fixed strategies means that memory is not a requirement for behavior.

On a two-dimensional lattice of varying sizes, interacting players face the following payoffs: $R = 1$, $T = b$ ($b > 1$), $S = P = 0$. The parameter b identifies the relative advantage of defection versus cooperation.[1] The entire lattice is occupied with C's or D's and again the scoring is the sum of encounters with neighbors. At the start of a next generation (a time-step) each cell is occupied by the player with the greatest score among the previous owner and the defined neighbors. The radius and the boundary conditions are varied. The rules among the n^2 players on the $n \times n$ lattice are deterministic. Updating of sites occurs in discrete time and in a synchronous fashion.

As reported by Nowak and May (1992), extremely complex patterns are achieved even when strategies are fixed that in essence involve no strategies at all. What they show is that ecological structure of cooperation stems from the magnitude of the advantage given to defecting, the value of b. For instance, the initial configuration starts with 10 percent defectors when b is between 1.75 and 1.8, indicating the prevalence of cooperation. When the b parameter is moved to values between 1.8 and 2, the lattice evolves. Nowak and May (1992, 827) interpret their results as indicating that C and D "coexist indefinitely in a chaotically shifting balance, with the frequency of C being (almost) completely independent of the initial conditions." This means that the asymptotic C fraction, f_c, roughly .318, remains constant for nearly all conditions tested. The inference is that of nonconvergence on defection commonly reported in iterative prisoner's dilemma games.

Despite the assertion of spatial chaos, the appearance of complex spatial patterns does not imply a Class III cellular automata. Again, Class III implies that disorder exists on both a local and global scale. In a game-theoretic situation, as devised by Nowak and May (1992), strategies are fixed. Because strategies are not available to each player (cell), the rigidity of the game may preclude deterministic chaos. Yet the difficulty of characterizing the patterns that emerge may well imply that the complexity found reflects the interesting Class IV behavior.

In a recent paper, Huberman and Glance (1993) examined Nowak and May's findings relaxing the assumptions of synchronous updates of sites. They report that the spatial complexity and richness of evolved actors disappear and the population moves quickly to converge on defection. Bersini and Detours (1993) also report that asynchrony induces significant stability and structure in cellular automata based models. Clearly, more analysis is needed.

Axiomatic Theory

Since the seminal work of Arrow (1963), the indeterminate characteristic of politics in majority-rule settings has caused a search for political equilibrium. Indeed, democratic theory once could only be understood with the proviso that neither global nor local equilibrium could exist within even the simplest

L. Douglas Kiel and Euel Elliott, Editors
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multidimensional voting system. Any outcome of any majority-rule system could cycle among alternatives at any time (McKelvey 1976, 1979; Schofield 1978; Cohen 1979). Any status quo could be overthrown by some other alternative drawn randomly from the entire issue space (Riker 1982). Even in political settings packed with elites, such as legislatures, decision making would be in constant disarray; policy decisions would be utterly unpredictable. Governments based on populist democratic voting theory should be without order or structure.

Human subject experimental work on the spatial properties of voting systems found outcomes clustered around a central tendency (Fiorina and Plott 1978). In experimental voting games where any outcome was feasible, different groups converged on similar collective choices rather quickly. So while the presence of political disequilibrium was well known, the microscopic causes and dynamics that might induce political order within multidimensional voting environments or disorder were not.

Serious progress has been made. Using simulation techniques, Browne, James, and Miller (1991, 1992) have proven the link between cyclical behavior under conditions of majority rule. In related work, Richards (1991, 1992) has formally linked the Li and Yorke (1975) period-3 theorem to disequilibrium in multidimensional choice. Both efforts focus on the dynamical nature of decision makers and environments and the interaction that follows proposals to alter the status quo.

In their initial work, Browne, James, and Miller (1991) confirmed that the McKelvey-Schofield-Cohen models exhibited deterministic chaos. It was by no means clear that the "chaos" confirmed by axiomatic theories of choice was comparable to dynamical chaos. Using three-dimensional, noncooperative fixed ideal points in smooth and continuous policy space, they proved that the position of the status quo is iterated by decision makers attempting to maximize expected utility based on their own preferred positions. Their resulting analysis shows a Lyapunov-positive dynamical system (see Browne, James, and Miller 1991).

Richards (1991) employs the Li-Yorke period-3 cycle theorem to prove the link between chaos and conditions in multidimensional choice under conditions of majority rule. The Li-Yorke theorem is an extension of the Sarkovskii theorem (1964), which states: If $f(x)$ has a point x that leads to a cycle of period π, then it also has a point x' that leads to a τ-cycle for every $\tau \leftarrow \pi$. Here, τ and π are elements of the following sequence:

$$
\begin{aligned}
&1 \leftarrow 2 \leftarrow 4 \leftarrow 8 \leftarrow 16 \ldots 2^m \ldots \leftarrow \\
&\ldots 2^m \cdot 9 \leftarrow 2^m \cdot 7 \leftarrow 2^m \cdot 5 \leftarrow 2^m \cdot 3 \ldots \leftarrow \\
&\ldots 2^2 \cdot 9 \leftarrow 2^2 \cdot 7 \leftarrow 2^2 \cdot 5 \leftarrow 2^2 \cdot 3 \ldots \leftarrow \\
&\ldots 2 \cdot 9 \leftarrow 2 \cdot 7 \leftarrow 2 \cdot 5 \leftarrow 2 \cdot 3 \ldots \leftarrow \\
&\ldots 9 \leftarrow 7 \leftarrow 5 \leftarrow 3.
\end{aligned}
$$

(As before, "←" means "precedes.") If $f(x)$ has period-3, then it must include all periods n when n is an arbitrary integer. This inclusion for $f(x)$ must also contain a chaotic region. By definition, the emergence of period-3 implies three frequencies: f^1, f^2, and f^3, where $f^3 = f^2/f^1$ (Falconer 1990).[2] Period-3 is one example where chaos was predicted from a mathematical theorem but could not be seen in earlier computer simulations (Mayer-Kress and Haken 1981).

Richards restates the Li-Yorke Theorem, sketches a proof, and links it to the three-cycle for the majority-rule function. Written without the work of Browne, James, and Miller (1991) at hand, Richards clearly delimits and urges the analysis they pursue. That is, based on the presence of a strange attractor, Richards suggests that within the dynamical decision process, a new solution set is likely to emerge. In a sequel, Richards (1992) extends the logic of the analysis considerably by linking the presence of period-3 cycles to multidimensional social choice situations in which the core is empty. Without testing numerically for the presence of chaos, Richards indicates with a series of bifurcation diagrams the fractal quality of the complex decision behavior over increasingly smaller scales.

Unraveling the dynamics of axiomatic theories under conditions of majority rule has presented some interesting analytic problems. In their highly innovative work, James, Browne, and Miller (1993) are reconsidering the dynamics of both individual decision makers and resultant system response. Policy space is allowed to automatically initiate system preserving reactions as a result of changes in a decision maker's behavior.

Both proposals, to move the status quo and acceptance of the change, are allowed and indeed expected to have an effect on a policy space surface. When a decision maker proposes or changes a policy position, the system responds in an effort to automatically reestablish the previous alignment. This systemic action may cause local disturbances, and the resulting ripple through the adjacent regions of policy space is allowed to produce a changed surface. The decision maker, as a consequence, faces an altered policy environment at the next iteration. These innovations allow for what we expect to be an "institutional response of entrenched interests against significant alterations of the system status quo. The expected consequence of such resistance would be a reordering of positions in the vicinity of a changed status quo that, at the same time, preserves the general stability of existing relationships" (Brown et al. 1992, 13). The inability to preserve such stability could lead to greater oscillations in support and to behaviors that may cause the system to go extinct. Because systems do respond, such events are likely to be rare.

Ideal point dynamics are comparably developed. Choice theory assumes that once a position is taken, it doesn't change. This assumption limits resulting cyclical behavior to a subregion of policy space. James et al. (1993) relax this constraint and allow the ideal points to move within policy space condi-

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tional to the iterated status quo locations. As the status quo iterates in policy space, the change in the position of the status quo indicates the degree of stress that must exist on the surface of the policy space. These forces constitute the sum of the moments impacting a subregion in policy space; their limits symbolize the critical values for phase transition.

At the micro-level, James et al. (1993) provide a decision maker with a changed context within which to reassess what might maximize his or her utility. Hence, the movement of the ideal point is not random in issue space but dependent upon the change in the immediate local field of forces, a very old electoral behavior idea brought to bear on behavior in multidimensional choice environments.

At the onset of cycling behavior, two things must occur if the system is in a chaotic state: (1) there is a rapid diminishing of information on where the next status quo is likely to be and (2) there is a dilemma as to what to do next. Again, Browne, James, and Miller state: " . . . continuous cycling is likely to produce strain in the region of the policy space, and rather than accept this state, actors (especially those whose ideal points are most distant from the cycling location) have an incentive to break the cycle by relocating their ideal points" (Browne et al. 1991b, 13). Implicitly, they are shifting the meaning of ideal points away from the strict rational-choice definition to a relative preferred policy position based on what it is currently possible to realize.

In any phase transition, for instance, any smoothed approximation, or envelope, of the volume occupied by area such as a status quo will grow in volume as it is moved. Unperturbed, the separatrices divide oscillatory and rotational trajectories of the various contours in utility space. Utility space, of course, exists as a projection of indifference functions. As the utility space is perturbed, a stochastic layer could form in its vicinity. What Browne, James, and Miller show is that the region occupied by the stochastic layer contains chaotic motion and holes that a stochastic trajectory cannot enter. This stochastic layer, moreover, is not uniformly thick, and the topology of the augmented space is therefore assumed to be coarse-grained or irregular.

Chaos as Allegory

Maybe the revolution caused by quantum theory during the first half of this century swamped Poincaré's work. Maybe the nature of nonlinearities would have been impossible to see without the development of high-end computers and the graphical ability to visualize complex mathematics. But it probably took serious applications or even potential applications to move developments like chaos theory off the pile of intellectual toys. Clearly, the characterization of complex dynamics and chaos with low degrees of freedom have opened up a scientific reality in chemistry, physics, and biology. But what is reasonable for chemistry, physics, or neurobiology is more difficult to accept in the social sciences.

It is difficult to find crisp illustrations of chaos in politics. They are rare precisely because politics is characteristically defined and affected by interactions among individuals, between individuals and groups, among groups, and so on. Political reality is often shaped by larger metapolitical environments. Thus, the dynamics and interactions that are the source of political chaos may obscure its presence.

A science of politics, like other quantitative sciences, may one day soon begin to reconcile itself to the fact that not everything important to understanding politics is accessible to traditional methods of data collection and statistical analysis. Some might say it is unfair to expect prediction and accurate forecasting because politics has been systematically investigated for too brief a time. Compared to astronomy, in which observations have taken place for thousands of years, or even demography, for hundreds of years, the politics of mass publics, elites, or even the behavior of political business cycles, have been studied only for decades. Some would suggest we have been too quick to attempt to generalize based on data over a short period of time and a limited range of political situations and contexts. Maybe. Others might say that while a time series would be useful, what we really need is a better understanding of the formal nature of political decision making. That is, we have not reduced the individual and the logic of the collective decision to a small enough package of variables to really understand the most basic political motives and actions. But as we have already suggested, even formal areas of political theory such as collective choice are now under investigation by nonlinear science.

Systems with strong nonlinearities, which we showed above to exist in relatively simple polities, and the possible chaos that results do present a practical problem for continuing conventional techniques of data analysis. In potentially chaotic systems, as several of the chapters in this book show, the observed variation is a mixture of real noise or stochastic error and deterministic chaos. May (1989) says a problem of nonlinear relations is separating density-dependent noise from density-independent noise. In some patches in a fluctuating environment, nonlinear relations are sufficient to produce chaos. The odiousness of the world is also able to generate noise by environmental stochasticity.

When or where chaos lurks in politics probably can only be known by statistical prediction. The beauty of recent developments in the theory of chaotic behavior is that they offer a wealth of predictions, given that we know the dynamics of the system (equations of motion). Research in areas such as cellular automata may begin to give us such equations for politics. Developments also provide powerful methods to extract the characteristics of chaotic dynamics (attractors, invariant measures, characteristic exponents) from data when we do not know the underlying dynamics.

At a minimum, the theory of chaos warns us of the dangers of extrapolation from a political time series. But the questions raised (and yet unanswered) are profound: Can we predict that a political system will display deterministic chaos? Can we link our current statistical methods and data to the existence of deterministic chaos? And does the existence of deterministic chaos imply that predictability in politics is impossible, or can the analyst learn from it and refine predictive models to ultimately reflect control over political processes?

Implicit and explicit assumptions exist about the political link between the micro- and macro-levels. For instance, some believe that a direct, simple, additive relationship exists between micro traits and macro manifestations. In the earlier work in electoral behavior, the level of an individual's partisan strength was assumed, when aggregated, to indicate the strength of the party system; the summation of individual identification was assumed to give a clear indication of the standing vote of the citizenry, and so on. Such simple micro-macro links were thought to hold for a variety of measurable variables: ideology, trust in government, and partisanship. These measures indicated the state of the political system and gave the analyst a handle on the immediate nature of the electoral system, as well as an ability to assess the long-term nature of the entire political system.

Nonlinear theory involves an approach quite different from conventional methods in quantitative political science. Conceptually, it shifts the emphasis in formal theory to investigating dynamical microscopic interactive behavior. At the macroscopic level, the emphasis is shifted from short-term forecasting based on specific, complete initial data to long-term predictions based on generic, incomplete data. Explicit theory relies upon elusive contextual variables like political climate or mood, level of political stimulation, instability, hegemony, or volatility. These variables are ill-defined as aggregation of individual attitudes but may be well defined in terms of certain statistical properties of the system.

Beyond Chaos

The purpose of theory is to make nature stand still when our backs are turned, Einstein reportedly said. For political scientists, nature often laughs and dances around behind us. Is the dance chaotic? In other disciplines, scientists are finding that chaos is a normal part of any process. Standing still—order, stability, and continuity—is merely a special case within a more complex process.

Some political mechanics govern electoral realignment, equilibrium between parties, the collapse of the Soviet Union or the return to order in Lebanon, the occurrence of a coup d'état, and so on. While politics is a complicated process, its complexity may even be understandable and manageable once political behavior can be precisely described. Yet despite the wealth of contemporary data and information, we really do not know how to precisely describe political behavior. We do not know the exact degrees of freedom or the equation(s) of motion that describe (and maybe predict and control) political dynamics. We don't know which political phenomena are linear and hence if they can be Fourier transformed.

Do equations that describe the cyclical nature of electoral politics also describe other political dynamics? Do political dynamical equations contain universal features similar to other social, economic, or biological dynamics? To borrow and to twist a line from Feynman's *Lectures on Physics*, political scientists cannot say whether something beyond these equations, like free will or personal liberty, is even needed. Recent discoveries in nonlinear dynamics may help us to eventually discover political laws and their qualitative content.

NOTES

1. Mutual cooperators therefore score 1; mutual defectors score 0; D scores b against C, who receives 0. The reported results are not altered when $P = \varepsilon$, when $0 < \varepsilon \ll 0$. $T > R > P > S$ remains strictly satisfied.
2. Period-3 means that $f^3(x_0) \leq x_0 < f(x_0) < f^2(x_0)$.