

CHAPTER 7

The Prediction of Unpredictability: Applications of the New Paradigm of Chaos in Dynamical Systems to the Old Problem of the Stability of a System of Hostile Nations

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The social sciences have long tried to emulate the procedures and results of the physical sciences—the laws of economic determinism of Marx come immediately to mind. To a large extent, the science of international relations has not been successful in this emulation; its capability of predicting the outcome of international competition (war or peace?) has been very limited. Perhaps this is because too many variables seem to be required, intuitively, to describe the interactions between nations. Also, nations are composed of multitudes of individually complex people, making it hard to believe that the international variables, representing their collective behaviors, can satisfy relatively simple functional relationships. Any easily expressible theory, relating such variables, is bound to be incomplete and thus suspect, whether the theory is expressed verbally or mathematically.

The physical sciences owe much of their long-term successful hold on the public imagination and pocketbook to their long record of successful predictions of physical events and processes, predictions that have given rise to our powerful modern technological society. In the process, the sciences have learned that incomplete descriptions of the systems being studied are still very useful. Of the many variables evidently needed to describe a physical system, some are major, more are minor, and often the latter can be ignored while still producing very useful predictions about the system. For example, celestial mechanics can give a very good predictive description of planetary motion in our solar system without including the effects of the motions of the various moons about their respective planets. The minor variables only lead to important effects in those regions of the system where the incomplete model, consisting only of the major variables, is itself unstable. Successful detailed predictions have usually been obtained by avoiding these unstable regions,

regions in which small changes in the (major) variables included in the model, mirroring the effect of the excluded (minor) variables, lead to major changes in the system. Such instability, in a deterministic system, is now commonly referred to as *chaos*.

Physics also deals with multitudes of complex entities, such as molecules. When appropriate—that is, when the interest is in the collection, for example, a gas, rather than in its individual constituent molecules—physics successfully uses very few variables. These are often the result of averaging over the constituent multitudes, satisfying relatively simple functional relationships, such as the gas laws and the fluid flow equations (e.g., Kinetic Theory, Fluid Physics; see Lerner and Trigg 1991). These laws are necessarily incomplete and hence sometimes incapable of leading to detailed predictions. But we have learned how to predict their unpredictability and have found that this incomplete knowledge is still useful as the model is extended to include further variables, that is, to more completeness.

Following the lead of physics, it will be assumed in this chapter that international relations may be productively modeled by relatively few variables, simply related; the choice of variables depends upon the system of interest and the questions to be answered. It will further be assumed that the breakdown of these incomplete models into chaos may be more useful than the models themselves, that the qualitative prediction of model instability may be more meaningful than the quantitative predictions of specific system variation.

Given these assumptions, a procedure is developed for exploring a number of questions that are of major concern to the understanding of international relations and the practical application of this understanding to the creation of international security policy. The procedure is the creation of simple mathematical models for the interaction of competing states and the examination of the numerical output of these models for regions of stability and chaos (Saperstein 1990). The general questions to be explored in this chapter are: “Which is more war-prone—a bipolar or a tripolar world?”; “Are democracies more or less prone to war than autocracies?”; and “Which is more war-prone—a system of shifting alliances or a collection of go-it-alone states?” (Further questions suitable for exploration with this procedure should become apparent to the reader.) For example, it will be shown that the region of instability in a tripolar world is larger than that of the corresponding bipolar world. From this it will be concluded that a tripolar world is more prone to war than a bipolar one.

Before applying these assumptions to the development of models and an analytical technique for getting useful information from these models, I will first try to justify them by attempting to formulate a general definition of science and prediction and then draw from this requirements for the more

specific science of international relations. An important part of this definition will be the relation between science and technology, a relation that has been fundamental to the growth of the natural sciences and their related technologies. The technology corresponding to the science of international relations should be the making of successful international security policy. Following the general discussion of prediction and nonpredictability, a tool for recognizing chaotic regions is developed. The chapter concludes with the development of a simple model for each question and the application of the chaos tool to the answering of that question.

Science (Understanding) and Technology (Control)

Many students in introductory courses in the sciences have great difficulty coming to grips with the notion of science as a system of understanding, not a compilation of facts or ideas about the system to be understood. Exacerbating the problem is the difficulty of compactly setting down what is universally meant by “understanding” in the physical or social sciences. By “understanding,” I mean the development of a relationship between the phenomenon to be understood and a previous set of concepts already understood (even if only innately). These prior concepts, axioms for this latest level of understanding, are similarly related, in turn, to previously understood axioms, which, in turn, are related to still more primitive understandings. Thus, science is a tower of relationships constantly growing at the top and the bottom.

By “relationship” between two levels of concepts or phenomena, I mean that one level can be derived from (caused by) the other level acting as axioms. “Derived,” in turn, means that the one follows logically from the other, arising by necessity from the structure of the language being used; there is a “functional relationship” between the two levels. For example, if the prior level consists of the two statements: “all men are mortal” and “Socrates is a man,” then the derived statement is “Socrates is a mortal.” More formally, if the system in question can be described by a variable (or set of variables), x , and if x_n and x_{n+1} represent two neighboring levels, then we might be able to write

$$x_{n+1} = f(x_n, \lambda), \quad (1)$$

where f designates the functional relation that may depend upon a set of parameters λ . In physical science this might be the relationship between the present configuration of planets in the solar system and a previous configuration. In biology, it might be the relation between the behavior of a cell in a salt solution to that of the same or similar cell in plain water. If there is a time sequence between the two levels—if one occurs at a later real time than the

other—then the implied later level is determined by the former. The resulting system of understanding—henceforth also referred to as “the theory”—is then a *deterministic system*. (An illustration of a deterministic system would be the sequence of positions along the orbit of an artillery shell in the absence of air resistance. A nondeterministic system would be the sequence of positions of an electron in an atom, though the probabilities of these positions are well determined by quantum physics.) For example, suppose the theory determines the value of some variable, x , at all future times, t , given the starting value of the variable, x_0 (this could be the result of iterating the relationship of equation 1 for all positive values of n starting with $n = 0$); the result of the theory is the curve $x(t)$ shown (see fig. 7.1, part a); $x(t)$ is determined by x_0 and the theory.

A check on the adequacy of the understanding system (the theory) is provided by observation of the system being understood. Are the axioms always observed to be accompanied by the derived concepts or phenomena (logical equivalence)? Or are they always followed by the next level? In this case, the consequences are predicted by the axioms. Such an empirical check requires confirmation of the validity of the axioms in the system under consideration, as well as the observation of their outcome. Hence, the prior level—the initial state of the system to be understood—must also be carefully observed. Actual measurements of the real world—either physical or social—are always accompanied by *noise*, random fluctuations in the values that would be obtained in a series of identical measurements upon identical systems (e.g., “error analysis” in Lerner and Trigg 1991). Hence there is always some uncertainty (also commonly called experimental error) in the observed system parameters or starting values, and hence some lack of certainty in the adequacy of the system of understanding. The result, therefore, is a range of possible starting values, δx_0 , and a consequent range in determined outcomes, $\delta x(t)$ (fig. 7.1, part b).

The phenomena being investigated may not refer to definite events but to the probability of occurrence of the events. For example, we may be investigating the probability of survival of a bacterium as an antibiotic is added to its growth medium. The events themselves may be random, nondeterministic, even though their probability is determined by the relations of the theory. In this case, single observations of the system, no matter how precise, are not sufficient. Repeated observations on an ensemble of identical systems (theoretically an infinite number of identical observations) are required to determine a probability (see, e.g., Feller 1950). For example, a single toss of a coin will not tell you the probability of a *head* for that coin. Nor will the observation of a single human life span give any actuarial probability information.

Thus a scientific theory relates some initial state (definite or a probability,

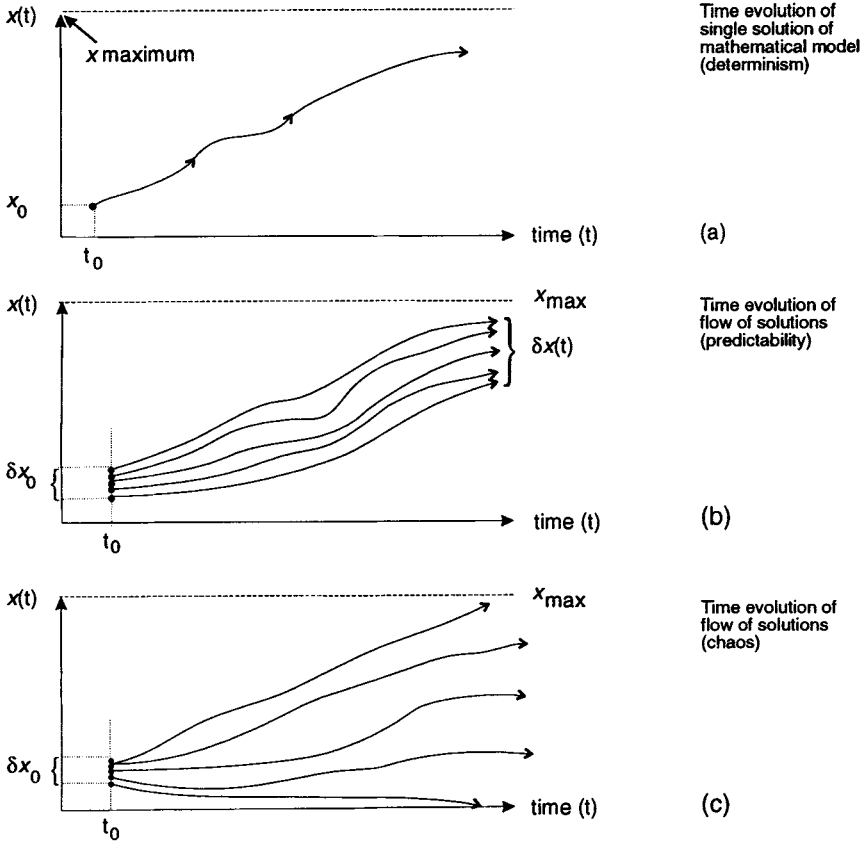


Fig. 7.1. Time evolution of model solutions under varying dynamics: (a) single solution of mathematical model (determinism); (b) flow of solutions (predictability); (c) flow of solutions (chaos).

but observed somehow) of the system to be understood to some other state(s) of the same system. Since the initial state is uncertain, to the extent of the observational *noise* encountered in empirically determining it, knowledge of the related states may also be uncertain. In *linear theories*, small changes in the parameters describing the initial state lead to correspondingly small changes in the related states. For example, if the theory is described by equation 1, then the small changes are similarly related:

$$x_{n+1} + \delta x_{n+1} = f(x_n + \delta x_n, \lambda). \quad (2)$$

Hence, given the theory, the parameters of the determined states are as well known as those of the initially given states (see fig. 7.1, part b). It follows that such a deterministic theory is also a predictive theory. The future state of the system is as well known as its present initial state. The ability of the physical sciences to make predictions, subsequently verified, has given them a strong hold on the human imagination. A popularly observed eclipse bolsters the public support of astronomy, as well as of all the other sciences, presumably applying the same methods. (Naturally enough, other intellectual and pseudointellectual disciplines thus strive to at least appear to be using the same scientific approaches.)

In *nonlinear theories*, differences between initial states may not evolve into allowed differences of final states; that is, equation 2 is not valid. Hence, small initial uncertainties about the system may lead to large final uncertainties; little or nothing may be known about the outcome, no matter how precise the knowledge of the initial state. For example, in figure 7.1, part c the range in outcomes determined by the theory is equal to the full range of values possible to the system; prediction may be impossible. If a small change in the initial configuration of a deterministic theory leads to large indeterminate changes in the output configurations, the theory is said to be *chaotic* (Schuster 1988); it precludes precise prediction. A given theory may be chaotic for some ranges of input or system parameters and nonchaotic and predictive for other possible conditions.

For example, the theory obtained by applying the usual Newtonian dynamical laws to the motion of physical fluids is often referred to as the Navier-Stokes equations. Given the initial state of, say, a body of water—the velocity of every particle of the water at some given starting time—the solutions of the equations will give the velocity of each of these particles at every subsequent time. Of course, noise and measurement errors are associated with the determination of the initial and system parameters (e.g., the configurations of the rocks in the creek bed over which the water flows; the random fluctuations of the molecules making up the fluid around their mean, fluid, motion). Hence, there will be fluctuations in the results produced by the equations. If the output fluctuations are small, the motion of the fluid is well predicted: knowledge of the behavior of the water at one place, at one time, gives knowledge of the fluid flow at neighboring places at subsequent times. Such a nonchaotic flow is termed *laminar flow* and is characteristic of liquids flowing slowly in smoothly varying channels. The same theory will describe the motion of the same water flowing rapidly in a boulder-strewn river, a flow that we usually describe as *turbulent* (see, e.g., Lerner and Trigg 1991). In this chaotic regime, knowledge of what the water is doing at one time and place provides almost no insight into what it will be doing later at the same and neighboring positions.

The difference between the laminar and turbulent regimes of the theory is determined by the value of a parameter, the Reynolds number, calculated from the fluid speed and the size of the obstructions to the fluid flow. For values of the Reynolds number less than a critical value, determined by the theory, the flow is laminar; exceeding the critical Reynolds number implies turbulence. The turbulent regime can further be divided into regimes of *soft chaos* and *hard chaos*. In the soft regime, the output fluctuations, though large with respect to the input parameter fluctuations, are small compared to the extent of the system. For example, there may be turbulent ripples on the surface of a body of water even though the motion of the bulk of the fluid is well predicted. In hard chaos the fluctuations dominate the entire system, as does the flow uncertainty in a fully turbulent, chaotically tumbling and swirling river rapids.

Although the flow of water in a turbulent stream may not be predictable, the onset of turbulence itself may be predicted. The transition from laminar flow to turbulent flow depends upon the Reynolds number, a parameter that changes as the stream is followed. Knowing how it changes, from a knowledge of the configuration of the stream bed and its sources, allows a prediction as to where and when the stream will change from laminar to turbulent flow and vice versa. Similarly, the transition to and from turbulent air flow over an aircraft's wings can be predicted as the shape of the aircraft and its aerial maneuvers are varied. Or, since a large storm may be construed as a turbulent part of the earth's atmosphere, the possibility of such a storm may be predicted even if the air flow within such a storm remains chaotic and unpredictable. Hence, a theory of a system that manifests considerable chaotic, and therefore unknowable, regimes, but that allows the prediction of where and when these chaotic regimes will start, still represents considerable understanding of that system. Such theories may not have the presumably all-powerful predictability of celestial mechanics, which has caught the public eye. But all natural systems may not be predictable (as we have learned in this century), and one of the characteristics of successful science is to do the best you can. The study of such unknowable systems has become very popular recently in the physical sciences (see, e.g., *chaos* in Lerner and Trigg 1991).

The possibility of prediction implies the possibility of deliberate control. Control of a process means that known intended consequences will follow from deliberate acts: prior conditions are modified to ensure desired outcomes or to avoid undesired outcomes. For example, if the outcome x_{n+1} in equation 1 is not desired, either the prior condition, x_n , or the system parameters, λ , can be changed appropriately. (If a car is going in a dangerous direction, the parameter λ —the steering wheel position—can be changed or the system can be started with the car on a different highway.) If prediction is not possible, there is no way of knowing the outcome of a given act or policy, which is

synonymous with saying control doesn't exist. Hence, technology, which is the controlled manipulation of some part of the human environment, requires predictability. A successful technology, one that produces anticipated ends in a system from given input policies and means, is thus a confirmation (though not a proof!) of the understanding of that system. Conversely, an unsuccessful technology, one in which anticipation is thwarted, is a definitive proof of the failure of the underlying theoretical understanding. The failure of a bridge or an agricultural policy is an indication that the appropriate understanding of the behavior of physical materials and environments or of farmers is lacking. The success of the bridge or policy, however, does not guarantee the correctness of the theoretical understanding. (There may be alternative theories to the one that maintains that guardian angels hold up the bridge and protect the crops.) It follows that there is a symbiotic relationship between the physical and biological sciences and engineering and medical technologies, a relationship that has existed ever since these sciences have existed. Scientific advances have led to new technologies that, in turn, "proved" the correctness of the scientific understanding and provided the tools for further measurements and theory. This symbiotic relationship has led to the success of both natural science and technology and has served as a model for all other sciences.

The Science of International Relations

Turning now to international relations, whose understanding is part of the political and social sciences, the corresponding technology is international policy making, with its subset, international security policies. If we truly understand the international system, we should be able to make successful policy, that is, policy whose desired outcome, given the existing system and starting configurations, actually occurs (see, e.g., Saperstein 1990). In the past, a successful international security policy meant that a nation obtained what it wanted, either without having to resort to war or by winning its wars without unreasonable costs to itself. That is, the international system, of which the nation is a part, either doesn't change to the detriment of the nation or, at the very least, the component of the system encompassing the nation thrives even if the rest of the system is fundamentally changed. In the present nuclear age, a war between major nuclear powers is unlikely to leave either the world system or any of its subcomponents intact. Hence, a major goal of any international policy has to be the avoidance of such a war. Thus, the symbiotic *science* must be able to predict the outbreak of war.

If that *science* could also predict the outcome of a war, there would be no need for the war. Nations often strive for overwhelming military strength in the hope of ensuring (predicting) their success in the event of war. They have often been unsuccessful in this prediction because of the large uncertainties in

the reactions of their potential opponents. Considering war as a chaotic process (see, e.g., Clausewitz 1982), like a turbulent flow over boulders, it may be possible to predict its outbreak but not its outcome—that is, which side of the boulder gets a given portion of churning water or which side wins the war.

A science of international relations that only dealt with probabilities could lead to no policy technology and hence would be useless. Testing of probabilistic arguments requires an ensemble of identical systems, whereas we only have one such system—our real world. Thus, a scientific conclusion that there is a likelihood of nuclear war will be treated as a prediction of certainty; the corresponding policy will almost certainly be avoided. In such a situation, probabilistic predictions such as “likely” or “unlikely” (where is the separation?) will be treated by policymakers as “yes” or “no,” which are the results of a definitive or deterministic science. Hence, we seek a science of international security with at least some deterministic aspects.

Can we postulate a system of competing, hostile states to be deterministic? Experience indicates that nations respond to one another’s actions in reasonably determined ways—at least for major actions and responses. If these patterns of action and response didn’t exist, there would be no political science, and perhaps no history, of international relations—just readings of tea leaves or goat’s entrails! But why should we expect deterministic behavior of a collection of nations, given that each one is composed of many, many individuals, each of whom often acts in an unpredictable, chaotic manner? An answer—that deterministic behavior of entities exists precisely because of the random nature of large numbers of their subentities—is obtained by analogy to the physicist’s description of a gas. Such a gas is a collection of an enormous number of randomly moving molecules whose precise description requires a similarly enormous number of stochastic variables. Yet its overall physical behavior is deterministically described by a few gas variables, such as pressure and temperature, each a complex average over the many molecular variables. On a gross scale of observation these few gas variables usually change smoothly, though on a very fine scale they exhibit random minor fluctuations, mirroring the underlying random molecular variables. So we might expect the behavior of modern nations to be governed by a relatively few deterministic variables, each a complex average over the behaviors of each of its multitudinous population. The usual goal is to find rules governing these variables, admitting that there will be occasional minor fluctuations—a love affair or assassination at the higher levels of government or society—not subject to these rules. My additional goal is to see how a system, evolving according to such rules, responds to such fluctuations.

The core of international relations can be described as follows: Nation A responds to the actions of nation B in the context of the other nations of the world system and its own present and past internal situations. Nation B then

responds to nation A's response in a similar context. Each response is determined by the system's previous responses (Richardson 1960a,b). Hence, the situation is recursively deterministic. If A and B designate the appropriate sets of variables describing the two nations, the situation might be represented in a manner similar to that of equation 1:

$$\begin{aligned} A_{n+1} &= a(A_n, B_n, \lambda) \\ B_{n+1} &= b(A_n, B_n, \lambda), \end{aligned} \quad (3)$$

where a and b are functional relationships representing the responses of the two nations; the parameters λ may also represent some average over the other nations in the world system. For example, A_n might represent the hostility (or the exports) of A to B in year n , with B_n similarly defined. Then equation 3 describes how next year's hostility (or exports) of A to B will be determined by this year's hostility (or exports) of A to B and of B to A. The role of the political scientists (and the need of the policymakers) is to attempt to predict the outcome of the intertwining chain of responses on the basis of their understanding of the system and its history. Usually this understanding is verbal, expressed in the words of historical and policy treatises. (In equation 3, the relationships a and b and the parameters λ would be verbally expressed.)

Verbal understanding possesses the vividness, vagueness, redundancy, and contradictions of ordinary, daily, spoken and written language. It usually contains many implicit assumptions and innate biases. The logic taking it from premise to conclusions is often incomplete and/or fuzzy. It is often difficult to know what the premises are and how well they are verified, what the conclusions are, and how we got from one to the other (e.g., the functions a and b in equation 3 are not explicit). Hence, how do you test the model against reality? In contrast, mathematical formalisms tend to be explicit in their premises, transparent in their logic, and concrete in their conclusions. There is thus an often-expressed desire to describe all aspects of the macroscopic interactive behavior of nations via explicit mathematical recursion relations between numerical variables representing those behaviors. Equation 3 becomes a set of mathematical relations between numerically defined variables: A , B , λ , and so on. If the resultant equations could be solved, given all the necessary input representing system parameters and starting conditions, they would produce concrete predictions that could be tested against observed behavior to confirm or negate the mathematical model.

However, given the complexity of the real-world system, we are unlikely to ever have all the necessary input information, or sufficient understanding, to create a model system of equations complete enough to really represent the world in all of its manifestations. Any practical model would have fewer

variables than are really required. The resultant equations, though far short of the complexity of the world, would still be complicated and nonlinear if they are to represent some of the essential attributes of our strongly interacting real world, ruling out the possibility of simple, explicit solutions (see, e.g., Prigogine and Stenger 1984). These obvious shortcomings of any mathematical model are no different than the difficulties of any reasonable verbal understanding of the world. The difference is that the incompleteness of the mathematical model is usually immediately obvious, whereas that of verbal models is concealed in their fuzzy verbosity. In both cases, the resultant predictions are necessarily incomplete, making theoretical testing and practical policy formulation difficult.

The more relevant factors left out of a model or incompletely described within the model, the more gross and incomplete its consequent predictions will be. Overall system variations may be correctly predicted though the detailed outcomes may not be trustworthy. Such incomplete predictions may or may not be useful, depending upon the circumstance of the policy making or theory building. It becomes important to determine what kind of gross predictions, from necessarily incomplete theoretical models, can be useful in a given arena of human experience. It is the contention of this chapter that the prediction of unpredictability can be very useful in attempts to understand and control the advent of war in the world system of nations (Saperstein 1984).

The Prediction of Chaos and the Outbreak of War

Crisis instability in the international system usually implies a configuration in which small insults can lead to major changes—the loss of a nail leads to the loss of a shoe . . . to the loss of a kingdom; an assassination of a minor duke can lead to the deaths of millions and the profound transformation of the world system. Such instability represents the loss of control and the great potential of war. This parallels the definition of *chaos* (Schuster 1988): small disturbances of a deterministic mathematical system lead to disproportionately large changes in the system and the consequent loss of control. Prediction of unpredictability in a system is a prediction of the onset of chaos—soft or hard. The range between soft and hard chaos among nations is the range between minor and major loss of control in international relations. It is the range between the possibility of loss of an unspecified soldier in a given battle, the winning or losing of that battle, of the war, and of the existent world system. We postulate that the presence of hard chaos, in a theoretical system modeling the international system of competing nations, is a representation of major crisis instability and of the extreme likelihood of the outbreak of major war in the real world being modeled. If the prediction of hard chaos is believable, then the international security policies associated with that part

of the model leading to the onset of chaos should be avoided. A useful analogy is to the testing of a new aircraft to determine its behavior under a wide variety of desirable and undesirable circumstances. If the theoretical model (either purely mathematical or based upon wind tunnel modeling) indicates that certain maneuvers are likely to lead to loss of control and possible loss of plane and crew, the pilots will be instructed to avoid those maneuvers—even if they would lead to otherwise desirable results. Even if the model is not sufficiently complete to indicate how the aircraft will behave in the danger zone but is believable in predicting the existence of the loss-of-control zone, that would be sufficient to mandate changing policy so as to stay away from that zone. Thus, if the prediction of unpredictability in an international system is believable, the ability to make such predictions, even if incomplete in details, can be used to answer political science questions as well as to help determine practical national security policy.

Thus we must face the question: is the prediction of unpredictability believable in an incomplete model? If the onset of chaos portends the outbreak of war, can we believe a model's prediction of war even though the extent and outcome of the war are not predicted and may not even be describable by the model? Is the prediction of instability (or stability) of a model itself stable in the face of expansions of the model to include more relevant aspects of the system? Will a model's prediction of chaos be softened or disappear as more variables and/or more complicated interactions are added to the model?

A formal answer to the above question requires a working knowledge of the complete model, knowledge that doesn't exist. (If it did, the question would be irrelevant!) In its place, we must rely on analogy with other systems—usually from mathematics or physics—where complete models, and their incomplete component models, exist and are used. Mathematical experience with dynamical systems indicates that “chaos first appears in the neighborhood of non-linear resonances” (Reichl 1992, 14), where new variables first make their impact upon the system. There is no experience of such chaotic regions disappearing as new variables are introduced. Empirical experience with fluids indicates that chaos (turbulence) appears earlier and stronger when new variables, such as temperature differences and heat flows, become important in the system. Theoretical experience with specific mathematical models of real phenomena (the Navier-Stokes equation for fluid flows; the recursive equations' modeling of the evolution of tripolar systems from bipolar ones [Saperstein 1991], developed later in this chapter) suggests that the regions of stability (areas of absence of chaos) decrease in extent when additional variables come into play.

We thus presume that qualitative (gross) predictions of the loss of model stability are much more believable (more stable) than are the quantitative (detailed) predictions from the same model. Hence, the prediction of hard

chaos in a model reasonably representing the international system of competing nations is a fair warning to policymakers: embarkation upon the modeled course of action is to be done with extreme dread and care. However, a contrary prediction of model stability is not an absolute assurance of system stability; complacent acceptance of the corresponding policy is not warranted since a more faithful, more complete model of the system in which the policy is being applied may very well allow chaotic breakdown. Similarly, when exploring theoretical questions about the international system, the appearance of chaos in an appropriate model portends the breakdown of stability in the system; however, stability of the model does not necessarily imply a corresponding stability in the system.

The stability of a mathematical system can be tested directly if sufficient computing power is available. For example, a large number of possible different but neighboring initial configurations of the system of interest may be specified. Each one of these starting configurations gives rise, via the mathematical model, to a final configuration (the system at some specified future time). If these predicted final states are as similar to each other as the initial states, the system is stable. If they are wildly divergent, as compared to the starting values, the system is chaotic. As an illustration, the SDI system proposed by President Reagan can be described by variables representing the number of offensive missiles available to the opponents, the number of defensive antimissile weapons, and the number of anti-anti-missile weapons (also offensive) (Saperstein and Mayer-Kress 1988). The model consists of relations between these variables specifying how each of the opponent parties procures weapons in response to the analogous procurements of the other nations in the system—a typical *arms race*. The initial configuration is specified by the initial numbers of the different weapons stocks and some numerical estimates of their capabilities: *neighboring* starting configurations imply slightly different values for these initial numbers. (In the real world, these numbers and parameters would not be precisely known to any of the competing parties.) The computer then tracks the evolution with time of the system emanating from each starting configuration and displays the variations in the possible outcomes—the numbers of the different types of weapons in the stocks of each of the opponents. If the final variations are comparable to the corresponding initial variations, the system is stable. If the final variations are large compared to the initial ones, but still small enough that system dominance (superior and inferior weapons stocks) can be distinguished, the system displays soft chaos. If the final outcomes vary so widely that they encompass the entire variation possible to the system and obscure all differences between the variables corresponding to the different nations, then no prediction of the outcome of the arms race is possible—we have hard chaos.

A more modest approach, suitable for systems with fewer variables, is

the calculation of *Lyapunov coefficients* for the model in question. These coefficients are direct measures of the rate at which initially neighboring configurations drift apart as the model system evolves. Each possible starting point traces out a path toward the future by means of the model mechanism, as is illustrated in figure 7.1. If the paths from closely neighboring starting points remain close (fig. 7.1b), prediction is possible; if the paths separate exponentially (fig. 7.1c), so that the final outcomes cover all possibilities allowed in the system, everything is possible in the future, meaning nothing is known about the future, and prediction is impossible—the situation is chaotic. The Lyapunov coefficient (see, e.g., Schuster 1988) is a useful measure of such path separation that is easily calculated given a computer able to follow the evolution of neighboring paths. Let X_0 and X'_0 be any two possible neighboring starting configurations, separated by a small distance δ_0 . In the course of time n these evolve respectively (e.g., via equation 1) to the two configurations X_n and X'_n , separated by the distance δ_n , where

$$\delta_n = \delta_0 e^{n\zeta(X_0)}. \quad (4)$$

This defines the Lyapunov coefficient $\zeta(X_0)$, which depends upon the starting configuration as well as the dynamics of the system. Since the interest is in the long-time evolution of the system, it follows that

$$\zeta(X_0) = \lim_{n \rightarrow \infty} \frac{\lim_{\delta_0 \rightarrow 0}}{\delta_0} \frac{1}{n} \log \left| \frac{X'_n - X_n}{\delta_0} \right|. \quad (5)$$

If $\zeta < 0$, two configurations starting close to each other will remain close to each other. Thus, predictability is possible (fig. 7.1b), so such a system will be defined as stable. If $\zeta > 0$, the initially close configurations will drift ever further apart, making prediction impossible. Thus $\zeta > 0$ is the signature of chaos or instability.

In this chapter, several aspects of international arms competitions, normally modeled qualitatively and verbally, will be put into equivalent algebraic forms. Only algebra is used! The variable expressing the arms level or procurement level of one country in one year is expressed in terms of similar variables describing the status of that country and of its competitor nations in the prior year (e.g., equation 3). These expressions only involve a small number of additions, subtractions, multiplications, and divisions. Hence, they can be put into standard spreadsheets on a desktop computer and iterated from year to year to generate the time evolution of the armaments levels of the competing nations. Thus, the future of these levels, and of the arms competition among these states, is determined. Similarly, the corresponding Lyapunov coefficients, describing how the uncertainty associated with these levels, given that there are initial uncertainties, evolves with time, are also

computed. Of course, one doesn't reach limits of infinity or zero, as is required by the definition in equation 5, on a desktop computer. Instead, the iterations are run up to large values of n (i.e., 100) for varying small values of δ_0 .

The prediction by a mathematical model is meaningful only if the results are stable, that is, if small changes in the input and/or model parameters lead to corresponding small changes in the predicted time evolutions. Otherwise, the model's output is chaotic and indicative that the system being modeled is crisis-unstable and likely to undergo a phase transition from a competitive, but nonshooting, *cold war* to a *hot war*. Since the models are necessarily incomplete, their detailed predictions are not really believable, even if meaningful within the context of the model itself. But, as we have presumed before, the prediction of chaos is itself meaningful and believable. Using the desktop computer, the models are explored over different ranges of parameters, or different algebraic forms, to see which regions lead to stable solutions, which to chaos. Since different parameter ranges or algebraic forms represent different policy choices, we are thus able to predict which policies will be dangerous, in that they may lead to crisis-unstable situations.

We have briefly described one approach that can lead to specific policy choices (e.g., S.D.I. should not be developed if it will lead to a decrease in the range of stability of the international offensive nuclear warhead missile system). In the remainder of this chapter the *chaos approach* will be applied to three political theory questions: Are bipolar international systems more or less stable than corresponding tripolar systems? Is a system of democratic nations more or less stable than corresponding systems of autocratic states? Is a system of nations that strives for a balance of power via shifting coalitions of states more or less stable than one in which each nation individually seeks to balance the power of the others?

There is a great body of literature exploring the answers to these questions via qualitative verbal analysis, and no claims are made for new or surprising answers. However, since all of these theoretical approaches, qualitative and quantitative, are incomplete and hence not completely convincing, a juxtaposition of different approaches that lead to similar answers should add to our understanding of these questions and their applicability. A disagreement between the results of the different approaches should be a signal that understanding is still lacking.

Which Is More War-Prone—A Bipolar or a Tripolar World?

Is a bipolar world more or less stable than a tripolar world? As we leave a world configuration dangerously dominated by two nuclear superpowers for a world perhaps dominated by none, one, or many, it is important to gain

insight into whether we must be even more cautious or can be a little less timid in addressing the many nonmilitary security problems of a twenty-first-century world. One approach to this question is to compare the range of stability of a two-power system with that of a corresponding three-power world.

The variable chosen to describe national behavior in the interactive model of equation 3 is the *devotion* of a given nation to war preparation—the ratio of arms procurement (in the most general sense) in a given year to the gross national product of that nation in the same year. Of necessity, the numerical value of this variable must lie between zero and one. The bipolar world is then heuristically modeled by the simple nonlinear Richardson-type model represented by the coupled equations below

$$\begin{aligned} X_{n+1} &= 4aY_n(1 - Y_n) \\ Y_{n+1} &= 4bX_n(1 - X_n). \end{aligned} \quad (6)$$

X_n is the devotion of nation X to war in year n . The procurement of arms by X in the year $n + 1$ is in response to, and assumed to be proportional to, the previous year's devotion of its opponent, nation Y. The nonlinear term $(1 - Y_n)$ expresses the assumption that if the opponent's resources are stretched to the breaking point (i.e., his previous year's procurements were so great that they almost preempted his entire GNP, making it impossible for him to procure any more), there is no need to stretch any further this year, so that this year's procurements may be correspondingly diminished. This model is so simple that its region of stability may be analytically computed (Saperstein 1984) as a function of the model's proportionality parameters, a and b , which must also lie between zero and one so that the X_n, Y_n remain between these bounds. The resultant curve represents the critical relation between a and b ; the region above the curve, in which the two Lyapunov coefficients, $\zeta(X_0)$ and $\zeta(Y_0)$, each defined as in equation 5, are positive, is the model's chaotic region.

In equation 7 the model is extended to three nations (when $\varepsilon = 0$, equation 7 reduces to the previous two-nation model, equation 6):

$$\begin{aligned} X_{n+1} &= 4aY_n(1 - Y_n) + 4\varepsilon Z_n(1 - Z_n) \\ Y_{n+1} &= 4bX_n(1 - X_n) + 4\varepsilon CZ_n(1 - Z_n) \\ Z_{n+1} &= 4\varepsilon[X_n(1 - X_n) + CY_n(1 - Y_n)]. \end{aligned} \quad (7)$$

As ε increases from zero, the coupling between X and Y and the third nation, Z, becomes increasingly significant. Numerical computations of the three Lyapunov coefficients pertinent to this model, $\zeta(X_0)$, $\zeta(Y_0)$, and $\zeta(Z_0)$, made using simple spreadsheets on a small desktop computer (Saperstein

1991), indicate that the stability region decreases in area as ε increases, that is, as the third nation becomes more significant in the world system. Hence, the model suggests, in accord with a great deal of other scholarly evidence, that a tripolar world is more dangerous than a bipolar one.

Are Democracies More or Less Prone to War?

Given the propensity of many of the world's major democracies to aid and abet less-than-democratic governments elsewhere, it would be useful to know if such policies hinder or further the world peace that these democracies presumably seek and guard. Required first is a definition of democracy that is easy to quantify and fit into a model of competitive arms procurement, such as that of equation 6, suitably generalized.

It seems easiest to define democracy in terms of diffuseness of decision making. The larger the fraction of the population that has significant input into matters of peace and war, the more democratic the nation will be considered to be. With this definition, the numerical difference between democracy and autocracy will be based upon population-sampling theory. I assume that every citizen of nation 1 has some intrinsic feeling for the totality of its opponent nation 2, and conversely, ranging from complete confidence to extreme fear and loathing. Let a_1 be the "fear and loathing" coefficient for any one citizen of nation 1. Then the policy of nation 1 toward its opponent will be determined by $\langle a_1 \rangle$, the mean of a_1 over the class of citizens of 1 who are significant decision makers; $\langle a_2 \rangle$ is similarly defined. The fraction of the citizens belonging to this class will be small in an autocracy, large in a democracy.

Assume that there is a natural spread of values of the fear and loathing coefficient whose distribution is characteristic of humanity, not nationality. Each large nation will contain many people manifesting small values of the coefficient, intermediate values, and so on. Thus the mean, \bar{a} , of this coefficient, taken over an entire large population, is a human characteristic, independent of nation: $\bar{a}_1 = \bar{a}_2 = \bar{a}$. (Note that the known examples of extreme interethnic hostility usually occur in comparatively small groups.)

The decision makers of a nation are a sample of the entire nation. If it is a very large sample, means over this sample will be close in value to means over the entire population. Hence, for large democracies, $\langle a \rangle \cong \bar{a}$. Means over small samples may vary significantly from the means over the entire population. Thus, given a random set of autocratic nations, there will be some for which $\langle a \rangle$ greatly exceeds \bar{a} and some for which $\langle a \rangle$ is much less than \bar{a} . In a corresponding set of democratic nations, all will have intermediate values of $\langle a \rangle = \bar{a}$. Therefore, if a_{xy} is the effective fear and loathing coefficient of nation X for nation Y, if all nations in the set containing X and Y are large democracies, $a_{xy} = \bar{a}$, whereas if the set contains autocracies, a_{xy} can range from very small values to very large values.

The effective fear and loathing coefficient is taken to be the proportionality constant in a nonlinear Richardson-type arms procurement recursion relation. In a competitive situation between nations X and Y, the annual procurement of arms by X, ΔX , will be proportional to the amount by which X's arms stocks were exceeded by Y in the previous year: $Y_n - X_n$. (The proportionality constant will be taken to be a_{xy} ; the similar coefficient for Y's annual arms buildup is a_{yx} .) Procurement will also increase proportionally to the total strength of Y: Y_n ; if Y is very weak, it won't matter if Y is stronger than X; the more powerful Y is, the more damaging any discrepancy in power will be in any potential conflict. However, arms buildups cannot continue indefinitely. There will be economic constraints; nations can't procure more than the total economy will allow. If C_x is the maximum annual expenditure for arms possible by X, assume a smoothed economic cutoff function

$$\tilde{\Theta}(1 - \Delta X/C_x) = (1 - \Delta X/C_x) \Theta(1 - \Delta X/C_x), \quad (8)$$

where Θ is the unit step function ($\Theta(\eta) = 1$ for $\eta > 0$, $\Theta = 0$ otherwise).

If X's arms stocks exceed those of Y, then X can decrease his armaments. Assume that the arms build-down in any one year (ΔX negative) will be proportional to the amount by which X exceeds Y's strength in the previous year and also proportional to the total power of X: the more confident a nation is in its strength, the more it can afford to build down. The proportionality parameter for build-down is the *confidence coefficient*, which, for simplicity, is taken to be the inverse of the build-up *fear and loathing* coefficient. Finally, there are no economic constraints for build-down. (It is assumed that reconversion of the military-industrial complex works!)

The result of combining the coupled arms buildups and build-downs is the following set of recursion relations, relating the total arms stocks of X and Y in any year to the corresponding stocks in the previous year:

$$\begin{aligned} X_{n+1} &= X_n + a_{xy} Y_n (Y_n - X_n) \tilde{\Theta}(1 - [X_{n+1} - X_n]/C_x) \Theta(Y_n - X_n) \\ &\quad - \frac{1}{a_{xy}} X_n (X_n - Y_n) \Theta(X_n - Y_n) \\ Y_{n+1} &= Y_n + a_{yx} X_n (X_n - Y_n) \tilde{\Theta}(1 - [Y_{n+1} - Y_n]/C_y) \Theta(X_n - Y_n) \\ &\quad - \frac{1}{a_{yx}} Y_n (Y_n - X_n) \Theta(Y_n - X_n). \end{aligned} \quad (9)$$

These relations may be iterated, given arbitrary starting values X_0 and Y_0 at $n = 0$, again using standard spreadsheet routines on a desktop computer. Going on to large n ($n = 100$), the two Lyapunov coefficients can be calculated from equation 5. Typical results (Saperstein 1992a) are given in figure

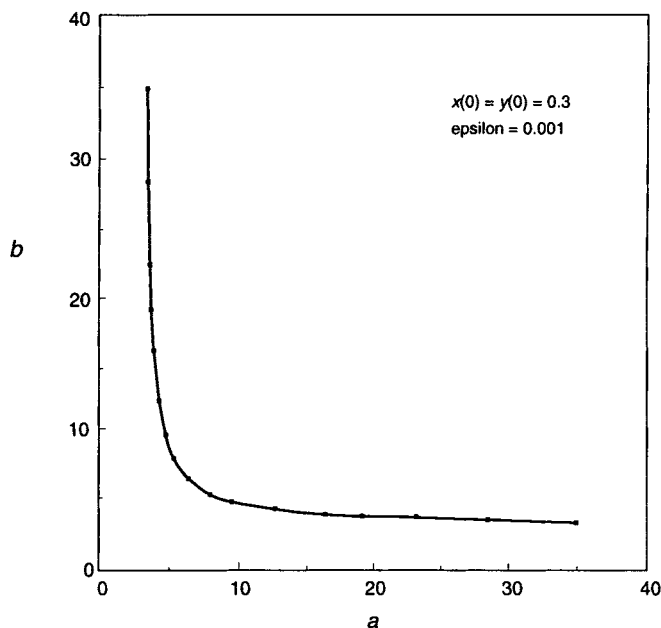


Fig. 7.2. Transition to escalating arms race

7.2, where the region above the curve is the region of instability. Note that small values for both $a = a_{xy}$ and $b = a_{yx}$ imply stability whereas instability results when both a_{xy} and a_{yx} get large or when either gets very large.

Since large values of the effective *fear and loathing* coefficient imply chaos and war, and since any collection of autocratic states (or large mixed collection of autocratic and democratic states) is more likely to have some pairs of nations with large values of this coefficient than a corresponding collection of democratic states, it follows that an outbreak of war is more likely in a collection of autocratic states than in a similar collection of democratic states. Even though it is often easier, in foreign affairs, to deal with autocratic governments, it is evident that, in addition to the usual human desire to nurture similar systems elsewhere, democracies are more likely to ensure world peace by supporting the democratization of other members of the international system.

Which is More War-Prone: A System of Shifting Alliances or a Collection of Go-It-Alone States?

As a last example, consider the question: which is a more stable system of competing states: one in which each nation individually procures arms so as to

match any other state in the system or one in which weaker nations join coalitions to collectively procure arms against the strong and in which they shift alliances when a coalition member becomes too strong—independence security or balance-of-power security? To model the two cases simply, consider three competing states, either independent of each other or in which any two will be allied against the stronger third.

Considering any pair of the three, arms buildup or build-down is presumed to occur just as in the previous case (equation 9), with a_{xy} the effective fear and loathing coefficient of X for Y and $1/a_{xy}$ the corresponding confidence coefficient. However, now X may pair against Y or Z, if all three are independent, so the resultant arms procurement or build-down is the sum of the corresponding pair terms:

$$\begin{aligned}\Delta X &= X_{n+1} - X_n = [a_{xy}Y_n(Y_n - X_n) \Theta(Y_n - X_n) + a_{xz}Z_n(Z_n - X_n) \\ &\quad \Theta(Z_n - X_n)] \tilde{\Theta}\left(\frac{1 - \Delta X}{C_x}\right) \\ &\quad - \frac{1}{a_{xy}}X_n(X_n - Y_n) \Theta(X_n - Y_n) - \frac{1}{a_{xz}}X_n(X_n - Z_n) \Theta(X_n - Z_n) \\ \Delta Y &= Y_{n+1} - Y_n = [a_{yx}X_n(X_n - Y_n) \Theta(X_n - Y_n) + a_{yz}Z_n(Z_n - Y_n) \\ &\quad \Theta(Z_n - Y_n)] \tilde{\Theta}\left(\frac{1 - \Delta Y}{C_y}\right) \\ &\quad - \frac{1}{a_{yx}}Y_n(Y_n - X_n) \Theta(Y_n - X_n) - \frac{1}{a_{yz}}Y_n(Y_n - Z_n) \Theta(Y_n - Z_n) \\ \Delta Z &= Z_{n+1} - Z_n = [a_{zx}X_n(X_n - Z_n) \Theta(X_n - Z_n) + a_{zy}Y_n(Y_n - Z_n) \\ &\quad \Theta(Y_n - Z_n)] \tilde{\Theta}\left(\frac{1 - \Delta Z}{C_z}\right) \\ &\quad - \frac{1}{a_{zx}}Z_n(Z_n - X_n) \Theta(Z_n - X_n) - \frac{1}{a_{zy}}Z_n(Z_n - Y_n) \Theta(Z_n - Y_n).\end{aligned}\tag{10}$$

In the alliance model, X will join its procurement with Y to match a superior Z, or join with Z to match a superior Y, and similarly for build-down (here the appropriate coefficients are written b_{xy} , b_{xz} , and so on):

$$\begin{aligned}\Delta X &= [b_{xy}Y_n(Y_n - X_n - Z_n) \Theta(Y_n - X_n - Z_n) + b_{xz}Z_n(Z_n - X_n - Y_n) \\ &\quad \Theta(Z_n - X_n - Y_n)] \tilde{\Theta}\left(\frac{1 - \Delta X}{C_x}\right)\end{aligned}$$

$$\begin{aligned}
 & -\frac{1}{b_{xy}}(X_n + Z_n)(X_n + Z_n - Y_n) \Theta(X_n + Z_n - Y_n) \\
 & -\frac{1}{b_{xz}}(X_n + Y_n)(X_n + Y_n - Z_n) \Theta(X_n + Y_n - Z_n) \\
 \Delta Y = & [b_{yx}X_n(X_n - Y_n - Z_n) \Theta(X_n - Y_n - Z_n) + b_{yz}Z_n(Z_n - Y_n - X_n) \\
 & \Theta(Z_n - Y_n - X_n)] \tilde{\Theta}\left(\frac{1 - \Delta Y}{C_y}\right) \\
 & -\frac{1}{b_{yx}}(Y_n + Z_n)(Y_n + Z_n - X_n) \Theta(Y_n + Z_n - X_n) \\
 & -\frac{1}{b_{yz}}(Y_n + X_n)(Y_n + X_n - Z_n) \Theta(Y_n + X_n - Z_n) \tag{11}
 \end{aligned}$$

$$\begin{aligned}
 \Delta Z = & [b_{zx}X_n(X_n - Z_n - Y_n) \Theta(X_n - Z_n - Y_n) + b_{zy}Y_n(Y_n - Z_n - X_n) \\
 & \Theta(Y_n - Z_n - X_n)] \tilde{\Theta}\left(\frac{1 - \Delta Z}{C_z}\right) \\
 & -\frac{1}{b_{zx}}(Z_n + Y_n)(Z_n + Y_n - X_n) \Theta(Z_n + Y_n - X_n) \\
 & -\frac{1}{b_{zy}}(Z_n + X_n)(Z_n + X_n - Y_n) \Theta(Z_n + X_n - Y_n).
 \end{aligned}$$

The two models each consist of three coupled recursion relations (eqs. 10 and 11). The answer to the question posed is obtained by seeing which of the two models is more stable.

The analysis is done by assuming that the initial values, X_0 , Y_0 , Z_0 , in each of the three nations are almost the same and asking whether these small differences grow as the recursion progresses. A spreadsheet iteration of the recursion relations is again carried out, starting off with X'_0 , Y'_0 , Z'_0 , representing small random differences about an arbitrarily chosen starting point, X_0 . Lyapunov coefficients are computed (again by iterating out to $n = 100$ with small δ_0), varying the starting value X_0 and the single model parameter a (representing a common value for a_{xy} , b_{xy} , a_{xz} , and so on).

Four possibilities become evident, as the parameters a and X_0 are varied, for the independent nation model as a result of the numerical computations (Saperstein 1992b). In the first case, *strong stability*, all sequence limits exist and all Lyapunov coefficients are negative, which implies that all sequence limits exist and are the same. There is complete predictability and hence no war. In the second case, *weak stability*, all sequences have limits that are

equal to each other, but the Lyapunov coefficients are positive. This occurs because all limits are less than the starting value, X_0 , except for the singular case when all start off exactly equal to X_0 , in which case the limiting value is X_0 . Hence, the numerator in the definition of the Lyapunov coefficient doesn't become less than the denominator, implying a positive coefficient. This situation is an illustration of *bifurcation*, in that one point leads to results completely different from all other points. Since the limits are still well defined, the sequences are predictable, not chaotic, and so again there is no war. In the third case, *weak chaos*, none of the sequence limits exist and so there are no Lyapunov coefficients. However, the variation within any one of the sequences, X'_n , Y'_n , Z'_n , and the differences between the sequences ($X'_n - X_n$) remain small compared to the arbitrary joint starting value, X_0 . The situation resembles case b in figure 7.1, rather than case c; hence, the unpredictability is minor and so, in the paradigm of this chapter, does not represent war. The last possibility is one in which the sequences do not have limits and the variations within each sequence, as well as the differences between them, are large compared to the starting value. In this case the range of unpredictability is the entire range possible, there is no way of knowing—even approximately—the outcome of any policy or action, and hence major fluctuations may result from minor perturbations. This is the criterion for crisis instability and war.

The numerical results for the independent nation model are displayed in figure 7.3. No sharp boundary was evident between the regimes of weak stability and weak chaos and so the two regimes are considered to be a single realm in which crisis stability is still valid. Note that there is no upper limit to the weak chaos regime, no breakdown of crisis instability, if the fear and loathing coefficient is small enough ($a \leq 1$). If there is enough confidence in the system, perturbations to the independent nation international system may lead to continued fluctuations and uncertainty, but not to war, no matter how large the initial, equal, stocks of arms are. On the other hand, with sufficient fear and loathing ($a > 1$), war will always break out if the initial arms stocks are large enough.

The numerical predictions of the alliance model are quite different. For large enough symmetric starting values, $X_0 = Y_0 = Z_0$, the system immediately and completely disarms, no matter what value is chosen for the fear and loathing coefficient. This is region I of figure 7.4; since all sequences tend immediately to a zero limiting value, it is a strong stability region. In region II of the figure, a limit has not yet been reached numerically at $n = 100$; nonetheless, the fluctuations in the monotonically decreasing sequences are small and hence this region is appropriately characterized as *weak chaos*: the course of events is clear and the eventual complete disarmament is completely predictable.

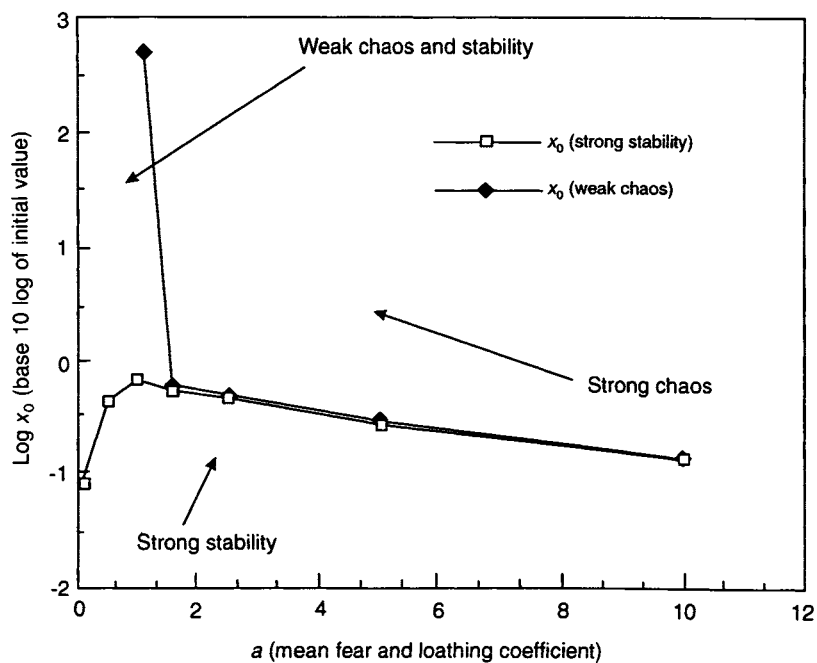


Fig. 7.3. Numerical prediction for the independent nation model

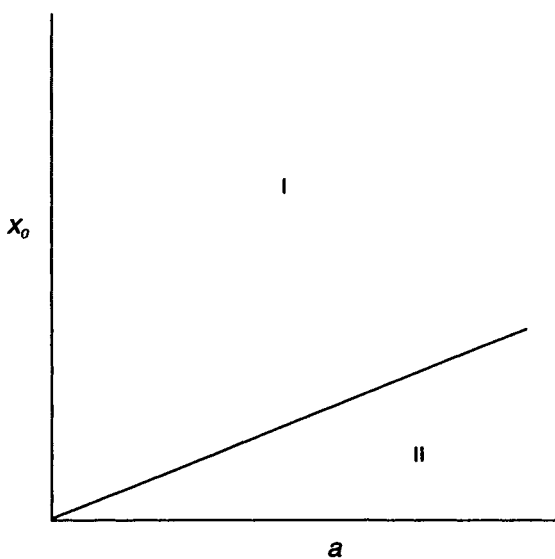


Fig. 7.4. Numerical prediction for the alliance model

Hence, using the possibility of strong chaos in the model as an indicator of the possibility of war in the corresponding international system, it follows that a policy of building (perhaps shifting) alliances is guaranteed to keep an initially symmetric system peaceful. A policy of seeking national security by “going it alone” allows the possibility of war in such a system if the contending parties start out with large enough levels of armed force.

I know of no real-world situation in which a system of comparable competing states have balanced themselves down to a complete disarmament as is predicted by the alliance model of this chapter. Perhaps this is because a shifting of alliances has costs in the real world, costs not included in this model. Also possible is that there are no real balance-of-power situations; real nations may resort to balance-of-threat instead, a much harder situation to model (Walt 1987). Or, the presumed anarchy may not have always been present.

Conclusion

Humans have always developed theories (often mythologies) to explain their world. With the advent of science, theories had to have practical as well as literary or philosophical implications. A successful theory of international relations should be an important component of competent policy making in the arena of international security in a competitive world. Conversely, a careful analysis of practical policy—successful or unsuccessful—must be the foundation for any scientific theory.

Given the complexity of the real international system, any theory of it is bound to be incomplete, describing some aspects of it more thoroughly than others. A truly massive modeling enterprise—such as is done in economics with national input/output models or in meteorology with large-scale computer models of the atmosphere—may at best lead to very rough, short-term, quantitative agreements between predicted and observed experience. The quality of the results is often vastly disproportionate to the effort required to secure them. It thus seems useful to look for simple models and methods of model analysis that can lead to qualitative robust predictions.

One such approach is the prediction of unpredictability in simple non-linear models of the competitive interactions between rival states. It can be an important approach to policy and theoretical questions, given the fundamental assumption that such unpredictability may represent crisis instability and war in the international system. Given that policy questions are usually more specific and complex than more theoretical questions, we expect to require more complex models for the former, although they would still be considerably simpler than any intended to address quantitative questions. For example, very simple theoretical models, easily analyzed with spread sheets on a desk-

top computer, have allowed us to conclude that a tripolar world is less stable than a corresponding bipolar world, that a set of democratic nations is more likely to be stable than a similar set that includes autocracies, and that a group of nations attempting to guard their security via balance-of-power alliance formation is more stable than a group in which each member makes individual attempts to cope with other potentially hostile members. A bigger and more complex computer model indicates that a policy of introducing SDI into a situation similar to the recent cold war nuclear confrontation between the United States and the Soviet Union would likely dangerously destabilize such a world.

None of these theoretical or policy results is new. Similar conclusions have been reached before by many others using conventional verbal analyses. But the alternative route presented in this chapter—the qualitative analysis of simple quantitative models—leads very quickly to the answers without any hidden implicit assumptions, biases, or faulty logic. Finally, agreement of results obtained via traditional means and the methods developed here brings added credence to both approaches. Science is a fabric: its ability to cover the world depends upon the existence of many different fibers acting together to give it structure and strength.