APPENDIX A.
DIFFERENTIATION RULES

Here is a table of useful differentiation rules (for a more complete list of differentiation rules, we refer the reader to Kleppner and Ramsey 1985).

Let a, b, c = constants; x, z, w = variables; y = a function of some variable(s); $f(\ )$, $g(\ )$ = functions.

TABLE A1. Some Useful Differentiation Rules

Expression	$\partial y/\partial x$	Explanation	Example
$y = c$	$\partial c/\partial x = 0$	The derivative of a constant is zero.	$\partial 7/\partial x = 0$
$y = cz$	$\partial(cz)/\partial x = 0$	The derivative of a term that does not depend on x is zero.	$\partial(3z)/\partial x = 0$
$y = cx$	$\partial(cx)/\partial x = c$	The derivative of a term involving a linear coefficient and x is that coefficient.	$\partial(3x)/\partial x = 3$
$y = cx^a$	$\partial(cx^a)/\partial x = acx^{a-1}$	The derivative of a term involving a linear coefficient and x raised to the ath power is the product of a, c, and x raised to the $(a - 1)$ power.	$\partial(3x^5)/\partial x = 15x^4$
$y = cxz$	$\partial(cxz)/\partial x = cz$	The derivative of a term involving a linear coefficient, x, and another variable, z, is the product of the coefficient and the variable (we can treat the other variable as a constant with respect to x here).	$\partial(3xz)/\partial x = 3z$
$y = cxzw$	$\partial(cxzw)/\partial x = czw$	The result extends to higher order interactions, where again variables that are not a function of the variable with respect to which one is differentiating are fixed.	$\partial(3xzw)/\partial x = 3zw$

$y = \ln(x)$	$\partial(\ln(x))/\partial x = 1/x$	The derivative of a logged variable is the inverse of that variable.	$\partial(3\ln(x))/\partial x = 3/x$
$y = e^x$	$\partial(e^x)/\partial x = e^x$	The derivative of base *e* raised to a variable is base *e* raised to that variable.	$\partial(3e^x)/\partial x = 3e^x$
$y = b_0 + b_x x + b_z z + b_{xz} xz$	$\partial b_0/\partial x + \partial(b_x x)/\partial x + \partial(b_z z)/\partial x + \partial(b_{xz} xz)/\partial x = b_x + b_{xz} z$	The derivative of some linear-additive function equals the sum of the derivative of each of the terms.	$\partial(1 + 2x + 3z + 4xz)/\partial x = 2 + 4z$
$y = f(x) \times g(x)$	$\partial(f(x) \times g(x))/\partial x = \partial(f(x))/\partial x\ g(x) + \partial(g(x))/\partial x\ f(x)$	The derivative of the product of two functions equals the sum of derivative of the first function, multiplied by the undifferentiated second function; plus the derivative of the second function, multiplied by the undifferentiated first function.	$\partial((2x + 5) \times (3\ln(x)))/\partial x = \partial((2x + 5))/\partial x\ (3\ln(x)) + \partial(3\ln(x))/\partial x\ (2x + 5) = 2(3\ln(x)) + (3/x)(2x + 5)$
$y = f(g(x))$	$(df/dg) \times (dg/dx)$	This is the chain rule for nested functions.	$\partial((2(3\ln x) + 5))/\partial x = \partial((2(g) + 5))/\partial g \times \partial g/\partial x = 2 \times (3/x) = 6/x$
$F \equiv$ a cumulative probability function for the probability density function *f*.	$\partial F(x)/\partial x = f(x)$	The derivative of any cumulative probability function is the corresponding probability density function.	$\partial\Phi(x)/\partial x = \phi(x)$